**Ministry of Higher
Education and Scientific
Research
Al-Isra'a University College
Medical Instrumentation
Engineering Technologies
Stage: Third stage**

وزارة التعليم العالي
والبحث العلمي
كلية الاسراء الجامعة
قسم هندسة تقنيات الاجهزة الطبية
المرحلة: الثالثة

Digital Signals Processing

معالجة الاشارات الرقمية

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Al-Isra'a University College

Medical Instrumentation

Engineering Technologies

Subject: Digital Signals Processing

Lecture No.: Lectures One and Two

Lecture Name: Discrete Time Signals

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Overview

1-A - Target population:-

For students of third class

Department of Medical Instrumentation Eng. Techniques

1-B - Rationale:-

This unit introduces principles of the discrete time signals, types of signals and demonstrates some fundamental signals.

1-C - Central Idea:-

The major topics discussed in this unit are included in the following outline.

- DISCRETE-TIME SIGNALS
- Complex Sequences
- Some Fundamental Sequences
- Signal Duration
- Periodic and Aperiodic Sequences
- Symmetric Sequences
- Signal Decomposition

Performance Objectives:-

After studying the 1st modular unit, the student will be able to:-

1. Define Discrete Time Signals.

2. Know the types of Discrete Time signals.

3. Utilities of fundamental Signals.

Pre test:-

Circle the correct answer:-

1. Discrete time signal is a function of:-

    a- Integer-valued variable, n.

    b- Fractional variable, n.

    c- Time.

    d- Speed.

2. The independent variable n represents:

  a- Time   b- Distance

  c- Spatial coordinate   d- Any one of above

3. Discrete-time signals are often derived by:-

    a- Sound.

    b- Sampling a continuous-time signal.

    c- Digital signal.

    d- Random signal.

4. Discrete-time signal may be:-

  a- Complex valued.

  b- Real valued.

  c- Imaginary valued.

  d- Any of above.

Signals play an important role in our daily life. A signal is defined as any physical quantity that varies with time, space, or any other independent variable or variables. Examples of signals that we encounter frequently are speech, music, picture, and video signals.

Mathematically, we describe a signal as a function of one or more independent variables.

For example, the functions

$$s_1(t) = 5t$$
$$s_2(t) = 20t^2$$

Describes two signals, one that varies linearly with the independent variable t (time) and a second that varies quadratically with t.

$$s(x, y) = 3x + 2xy + 10y^2 \quad \text{Two independent variables.}$$

1. DISCRETE-TIME SIGNALS

In digital signal processing, signals are represented as sequences of numbers, called samples. A sample value of a typical discrete – time signal or sequence is denoted as *x(n)* with the argument n being an integer in the range - ∞ and ∞. It should be noted that *x(n)* is defined only for integer values of n. The graphical representation of a sequence *x(n)* with real – valued samples is illustrated in figure 1.1
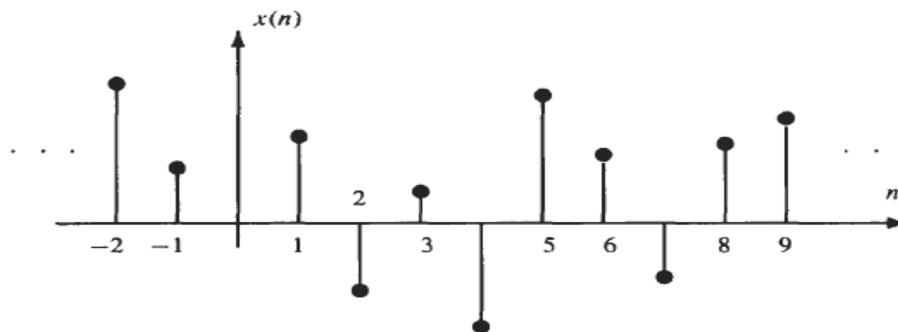
Fig. 1-1. The graphical representation of a discrete-time signal *x(n)*.

In some problems and applications it is convenient to view x(n) as a vector. Thus, the sequence values x(0) to x(N - 1) may often be considered to be the elements of a column vector as follows:

$$\mathbf{x} = [x(0),\ x(1),\ \dots,\ x(N-1)]^T$$

Discrete-time signals are often derived by sampling a continuous-time signal, such as speech, with an analog-to-digital (A/D) converter.' For example, a continuous-time signal $x_a(t)$ that is sampled at a rate of fs = l/Ts samples per second produces the sampled signal $x(n)$, which is related to $x_a(t)$ as follows:

$$x(n) = x_a(nT_s)$$

1.1 Complex Sequences

In general, a discrete-time signal may be complex-valued. In fact, in a number of important applications such as digital communications, complex signals arise naturally. A complex signal may be expressed either in term of its real and imaginary parts,

$$z(n) = a(n) + jb(n) = \text{Re}\{z(n)\} + j\text{Im}\{z(n)\}$$

or in polar form in terms of its magnitude and phase,

$$z(n) = |z(n)| \exp[j\arg\{z(n)\}]$$

The magnitude may be derived from the real and imaginary parts as follows:

$$|z(n)|^2 = \text{Re}^2\{z(n)\} + \text{Im}^2\{z(n)\}$$

Whereas the phase may be found using

$$\arg\{z(n)\} = \tan^{-1} \frac{\text{Im}\{z(n)\}}{\text{Re}\{z(n)\}}$$

If z(n) is a complex sequence, the complex conjugate, denoted by z*(n), is formed by changing the sign on the imaginary part of z(n):

$$z^*(n) = \text{Re}\{z(n)\} - j\,\text{Im}\{z(n)\} = |z(n)|\exp[-j\arg\{z(n)\}]$$

1.2 Some Fundamental Sequences

Although most information-bearing signals of practical interest are complicated functions of time, there are three simple, yet important, discrete-time signals that are frequently used in the representation and description of more complicated signals. These are the unit sample, the unit step, and the exponential. The unit sample, denoted by S(n), is defined by

$$\delta(n) = \begin{cases} 1 & n = 0 \\ 0 & \text{otherwise} \end{cases}$$

and plays the same role in discrete-time signal processing that the unit impulse plays in continuous-time signal processing. The unit step, denoted by u(n), is defined by

$$u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

and is related to the unit sample by

$$u(n) = \sum_{k=-\infty}^{n} \delta(k)$$

Similarly, a unit sample may be written as a difference of two steps:

$$\delta(n) = u(n) - u(n - 1)$$

Finally, an exponential sequence is defined by

$$x(n) = a^n$$

Where a may be a real or complex number. Of particular interest is the exponential sequence that is formed when a $=e^{j\omega_0}$, where ω_0, is a real number. In this case, x(n) is a complex exponential

$$e^{jn\omega_0} = \cos(n\omega_0) + j\sin(n\omega_0)$$

1.3 Signal Duration

Discrete-time signals may be conveniently classified in terms of their duration or extent. For example, a discrete time sequence is said to be a finite-length sequence if it is equal to zero for all values of n outside a finite interval [N1, N2]. Signals that are not finite in length, such as the unit step and the complex exponential, are said to be infinite-length sequences. Infinite-length sequences may further be classified as either being right-sided, left-sided, or two-sided. A right-sided sequence is any infinite-length sequence that is equal to zero for all values of n < n_o for some integer n_o. The unit step is an example of a right-sided sequence. Similarly, an infinite-length sequence *x(n)* is said to be left-sided if, for some integer n_o, *x(n)* = 0 for all n > n_o. An example of a left-sided sequence is

$$x(n) = u(n_0 - n) = \begin{cases} 1 & n \leq n_0 \\ 0 & n > n_0 \end{cases}$$

This is a time-reversed and delayed unit step. An infinite-length signal that is neither right-sided nor left-sided, such as the complex exponential, is referred to as a two-sided sequence.

1.4 Periodic and Aperiodic Sequences

A discrete-time signal may always be classified as either being periodic or aperiodic. A signal *x(n)* is said to be periodic if, for some positive real integer N,

$$x(n) = x(n + N) \tag{1.1}$$

For all n. This is equivalent to saying that the sequence repeats itself every N samples. If a signal is periodic with period N, it is also periodic with period 2N, period 3N, and all other integer multiples of N. The fundamental period, which we will denote by N, is the smallest positive integer for which Eq. (1.1) is satisfied. If Eq. (1 .1) is not satisfied for any integer N, x(n) is said to be an aperiodic signal.

EXAMPLE: The signals

$$x_1(n) = a^n u(n) = \begin{cases} a^n & n \geq 0 \\ 0 & n < 0 \end{cases}$$

$$x_2(n) = \cos(n^2)$$

are not periodic, whereas the signal

$$x_3(n) = e^{j\pi n/8}$$

is periodic and has a fundamental period of N = 16.

If x_1 (n) is a sequence that is periodic with a period N_1, and $x_2(n)$ is another sequence that is periodic with a period N_2, the sum

$$x(n) = x_1(n) + x_2(n)$$

will always be periodic and the fundamental period is

$$N = \frac{N_1 N_2}{\gcd(N_1, N_2)} \tag{1.2}$$

where gcd(NI, N2) means the greatest common divisor of N_1 and N_2. The same is true for the product; that is,

$$x(n) = x_1(n)x_2(n)$$

will be periodic with a period N given by Eq. (1.2). However, the fundamental period may be smaller.

Given any sequence *x(n),* a periodic signal may always be formed by replicating *x(n)* as follows:

$$y(n) = \sum_{k=-\infty}^{\infty} x(n - kN)$$

where N is a positive integer. In this case, *y(n)* will be periodic with period N.

1.5 Symmetric Sequences

A discrete-time signal will often possess some form of symmetry that may be exploited in solving problems. Two symmetries of interest are as follows:

Definition: A real-valued signal is said to be even if, for all n,

$$x(n) = x(-n)$$

whereas a signal is said to be odd if, for all n,

$$x(n) = -x(-n)$$

Any signal *x(n)* may be decomposed into a sum of its even part, $x_e(n),$ and its odd part, $x_o(n)$, as follows:

$$x(n) = x_e(n) + x_o(n) \qquad\qquad (1.3)$$

To find the even part of *x(n)* we form the sum

$$x_e(n) = \tfrac{1}{2}\{x(n) + x(-n)\}$$

whereas to find the odd part we take the difference

$$x_o(n) = \tfrac{1}{2}\{x(n) - x(-n)\}$$

For complex sequences the symmetries of interest are slightly different.

Definition: A complex signal is said to be conjugate symmetric if, for all n,

$$x(n) = x^*(-n)$$

and a signal is said to be conjugate anti symmetric if, for all n,

$$x(n) = -x^*(-n)$$

Any complex signal may always be decomposed into a sum of a conjugate symmetric signal and a conjugate anti symmetric signal.

1.6 Signal Decomposition

The unit sample may be used to decompose an arbitrary signal x(n) into a sum of weighted and shifted unit samples as follows:

$$x(n) = \cdots + x(-1)\delta(n+1) + x(0)\delta(n) + x(1)\delta(n-1) + x(2)\delta(n-2) + \cdots$$

This decomposition may be written concisely as

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\delta(n-k) \qquad (1.4)$$

where each term in the sum, *x(k)δ(n - k),* is a signal that has an amplitude of *x(k)* at time n = k and a value of zero for all other values of n. This decomposition is the discrete version of the shifting property for continuous-time signals and is used in the derivation of the convolution sum.

Post test:-

Circle the correct answer:-

1- A finite-length sequence:

a- Is equal to zero for all values of n outside a finite interval.

b- Is equal to one for all values of n outside a finite interval.

c- Is equal to one for all value of n.

d- Is equal to zero for all value of inside a finite interval.

2- A signal x(n) is said to be periodic if:

a- x(n) = x(n+2n).

b- x(n) = x(n+N).

c- x(n) = x(n+0.5n) .

d- x(n) = x(N).

3- An arbitrary signal x(n) can be decompose into sum of weighted and shifted:

a- Unit step.

b- Unit exponential.

c- Unit Impulse.

key answer:-

1- Pre test:-

1. **a**
2. **d**
3. **b**
4. **d**

2- Post test:-

1. **a**
2. **b**
3. **c**

References:-

1. Schaum's Outline of Theory and Problems of Digital Signal processing.

2. Digital signal processing, principles, algorithms, and applications by John G. Proakis and Dimitris G. Manolakis.

3. Signal and systems, Alan Oppenheim.