اسس كهرباء

قسم هندسة تقنيات التبريد والتكييف

المرحلة الاولى

المحاضرة الرابعة

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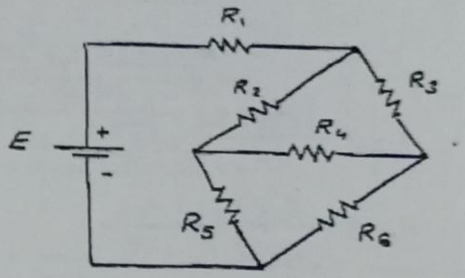
Wye - Delta Transformation

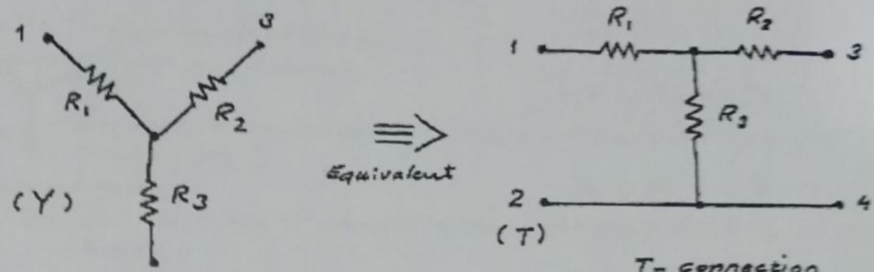
EE2

: There are some cases often arises in circuit analysis, when the resistors are neither in parallel nor in series. For example, consider the circuit shown :



In this circuit, R_1, R_2, R_3 ... R_6 are neither in series nor in parallel

* Wye (Y or star) connection

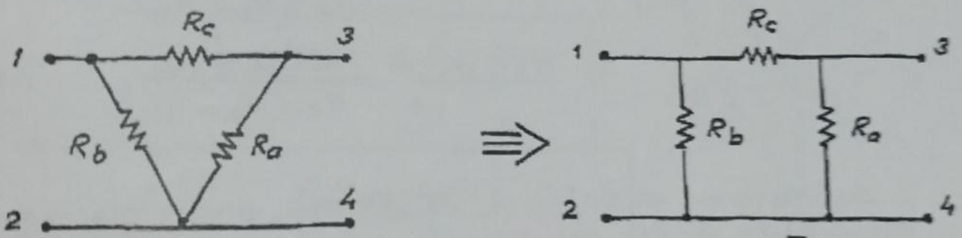


Y or star circuit connection

T- connection

Y and T connections

* Delta (Δ or π) circuit connection



Delta circuit connection & its equivalen Π- connection

Δ $\qquad$ Y

* Delta to Wye Transformation :

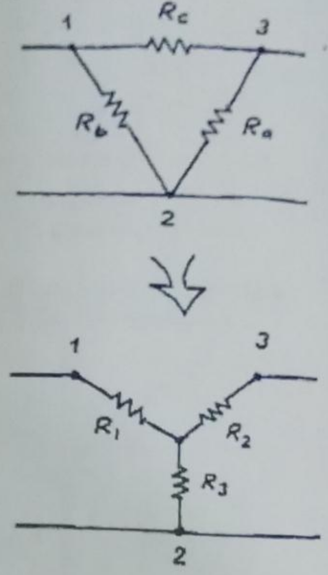
- We have Δ and want to get its equivalent star circuit

- Consider the Δ circuit shown; to be transformed into its equivalent star shown below :

$$R_{12} = \frac{R_b(R_a + R_c)}{R_a + R_b + R_c}$$

$$R_{13} = \frac{R_c(R_a + R_b)}{R_a + R_b + R_c}$$

$$R_{23} = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c}$$



$$R_{12} = R_1 + R_3$$

$$R_{13} = R_1 + R_2$$

$$R_{23} = R_2 + R_3$$

$$\therefore \quad R_1 + R_3 = \frac{R_b(R_a + R_c)}{R_a + R_b + R_c} \quad \cdots (1)$$

and

$$R_1 + R_2 = \frac{R_c(R_b + R_c)}{R_a + R_b + R_c} \quad \cdots (2)$$

$$R_2 + R_3 = \frac{R_a(R_b + R_c)}{R_a + R_b + R_c} \quad \cdots (3)$$

Subtraction Eq.(3) from Eq.(1) and adding the resulting equation to Eq.(1) results in :

$$\boxed{R_1 = \frac{R_b R_c}{R_a + R_b + R_c}}$$

Similarly ;

$$\boxed{R_2 = \frac{R_c R_a}{R_a + R_b + R_c}}$$

and;

$$\boxed{R_3 = \frac{R_a R_b}{R_a + R_b + R_c}}$$

IN GENERAL

> Each resistor in the Y network is the product of the resistors in the two adjacent Δ branches, divided by the sum of the three resistors

* Wye to Delta Transformation

- We have Y connected circuit and want to get its equivalent Δ.
- Consider the Y circuit shown, its equivalent Δ is shown below;

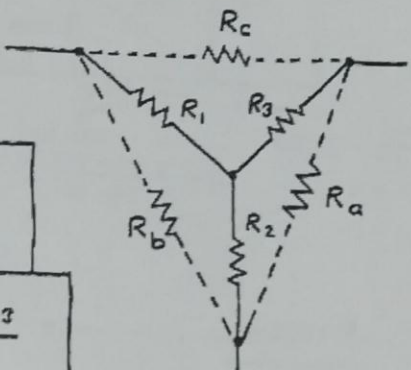
Using the previous sets of equations, then we have:

$$\boxed{R_a = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_1}}$$

$$\boxed{R_b = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_3}}$$

and

$$\boxed{R_c = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_2}}$$



IN GENERAL

> Each resistor in the Δ network is the sum of all possible products of Y resistors taken two at a time, divided by the opposite Y resistor.

Notes:

* The Y and Δ networks are said to be balanced when:

$$R_1 = R_2 = R_3 = R_Y$$

and

$$R_a = R_b = R_c = R_\Delta$$

* Under balance condition, the conversion equations become:

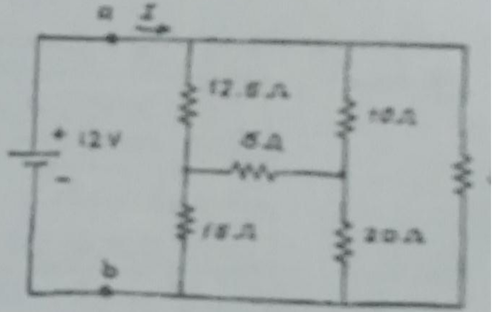
$$\boxed{R_Y = \frac{R_\Delta}{3}}$$

or

$$\boxed{R_\Delta = 3R_Y}$$

Example

Obtain the equivalent resistance R_{ab} for the circuit sh
and use it to find the current I



Solution

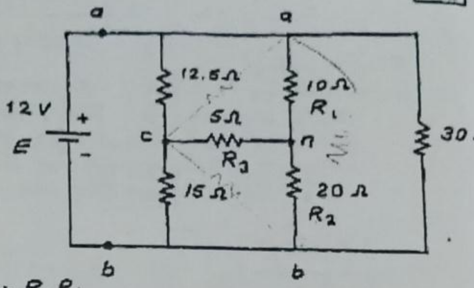
* We can't use the relations of series connected or parallel connected resistors to obtain R_{ab}.

* We try to use Δ-Y transformations or Y-Δ to get R_{ab}.

* If we transform the Y consisting of :

$R_1 = 10\,\Omega$
$R_2 = 20\,\Omega$
and $R_3 = 5\,\Omega$

$\therefore$ the equivalent Δ circuit contains :



$$R_a = \frac{R_1R_2 + R_2R_3 + R_1R_3}{R_1}$$

$$= \frac{10\times20 + 20\times5 + 5\times10}{10} = \frac{350}{10}$$

$$= 35\,\Omega$$

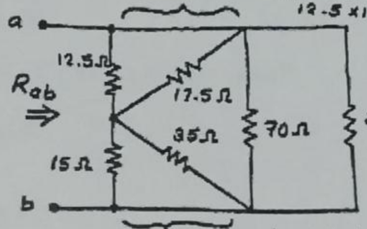
Similarly ;

$$R_b = \frac{R_1R_2 + R_2R_3 + R_1R_3}{20} = \frac{350}{20} = 17.5\,\Omega$$

and :

$$R_c = \frac{R_1R_2 + R_2R_3 + R_1R_3}{5} = \frac{350}{5} = 70\,\Omega$$

$12.5 // 17.5 = \frac{12.5\times17.5}{12.5\times17.5} = 7.3\,\Omega$



$15 // 35 = \frac{15\times35}{15+35} = 10\,\Omega$

$70 // 30 = \frac{70\times30}{70+30} = 21\,\Omega$

$$\therefore R_{ab} = (7.3 + 10.5) // 30$$

$$= \frac{17.8\times21}{17.8+21}$$

$$\Rightarrow R_{ab} = \underline{9.632\,\Omega}$$

$$\therefore I = \frac{E}{R_{eq}} = \frac{E}{R_{ab}}$$

$$= \frac{12}{9.632}$$

$$\Rightarrow I = \underline{1.246\ A}$$