Chapter – 6 Bearing Capacity and Settlement of Shallow Foundation

6.1. Introduction

Bearing capacity is the power of foundation soil to hold the forces from the superstructure without undergoing shear failure or excessive settlement. Foundation soil is that portion of ground which is subjected to additional stresses when foundation and superstructure are constructed on the ground.

Factors influencing Bearing Capacity:

Bearing capacity of soil depends on many factors. The following are some important ones.

1. Type of soil
2. Unit weight of soil
3. Surcharge load
4. Depth of foundation
5. Mode of failure
6. Size of footing
7. Shape of footing
8. Depth of water table
9. Eccentricity in footing load
10. Inclination of footing load
11. Inclination of ground
12. Inclination of base of foundation

The following are a few important terminologies related to bearing capacity of soil.

6.2. Basic definition and their relationship

❖ **Ultimate bearing capacity (q_u):**
The ultimate bearing capacity is the gross pressure at the base of the foundation at which soil fails inshear.

❖ **Net ultimate Bearing Capacity (q_{nu}):**
It is the net increase in pressure at the base of foundation that cause shear failure of the soil.

$q_{nu} = q_u - \gamma D_f$ (overburden pressure)

❖ **Net Safe Bearing Capacity (q_{ns}) :** It is the net soil pressure which can be safely applied to the soil considering only shear failure.

Thus, $q_{ns}=q_{nu}/FOS$ *(Factor of Safety usually taken as 2 to 3)*

❖ **Gross Safe Bearing Capacity (q_s) :**
It is the maximum pressure which the soil can carry safely without shear failure.

$q_s = q_{nu} / FOS + \gamma Df$

❖ **Net Safe Settlement Pressure (q_{np}) :**
It is the net pressure which the soil can carry without exceeding allowable settlement.

Net Allowable Bearing Pressure (q_{na}):
It is the net bearing pressure which can be used for design of foundation.

Thus,

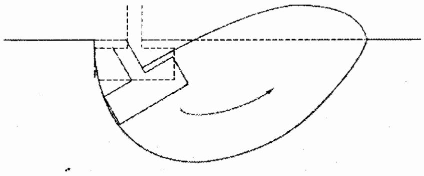
$q_{na} = q_{ns}$; if $q_{np} > q_{ns}$

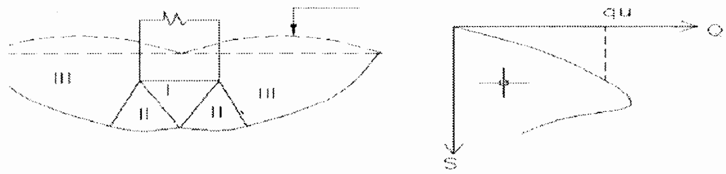
$q_{na} = q_{np}$; if $qns > q_{np}$

It is also known as Allowable Soil Pressure (ASP).

6.3. Principle mode of soil failure or Types of shear failure:

➢General shear failure: For Dense sand.

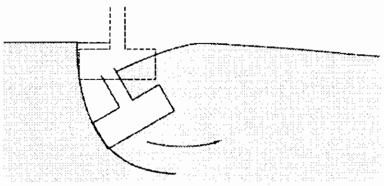


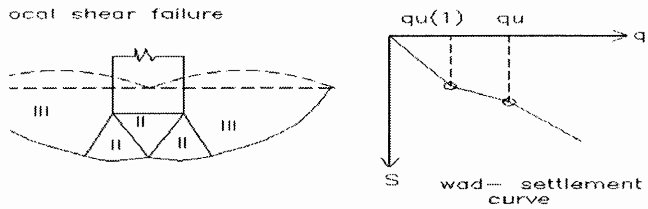


Strip footing resting on surface   Load –settlement curve of dense sand or stiff clay

- ❖ The load - Settlement curve in case of footing resting on surface of dense sand or stiff clays shows pronounced peak & failure occurs at very small stain.
- ❖ A loaded base on such soils sinks or tilts suddenly in to the ground showing a surface heave of adjoining soil
- ❖ The shearing strength is fully mobilized all along the slip surface & hence failure planes are well defined.
- ❖ The failure occurs at very small vertical strains accompanied by large lateral strains.
- ❖ $I_D > 65$,N>35, $\Phi > 36^0$, e < 0.55

➢Local shear failure: For Medium compaction soil.

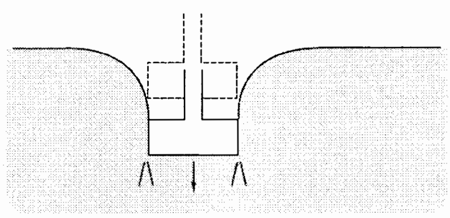


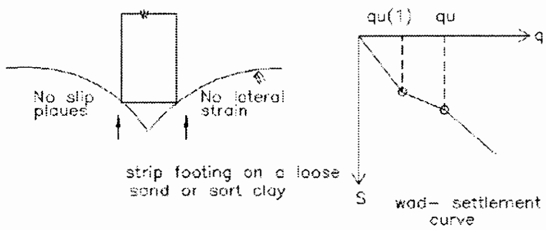


Strip footing resting on surface   Load –settlement curve of Medium sand or Medium clay when load is equal to a certain value qu(1),

- ❖ The foundation movement is accompanied by sudden jerks.
- ❖ The failure surface gradually extends out wards from the foundation.
- ❖ The failure starts at localized spot beneath the foundation & migrates out ward part by part gradually leading to ultimate failure.
- ❖ The shear strength of soil is not fully mobilized along planes & hence failure planes are not defined clearly.
- ❖ The failure occur at large vertical strain & very small lateral strains.
- ❖ I_D= 15 to 65 , N=10 to 30 , Φ <30, e>0.75

➢ Punching shear failure: For loose soil





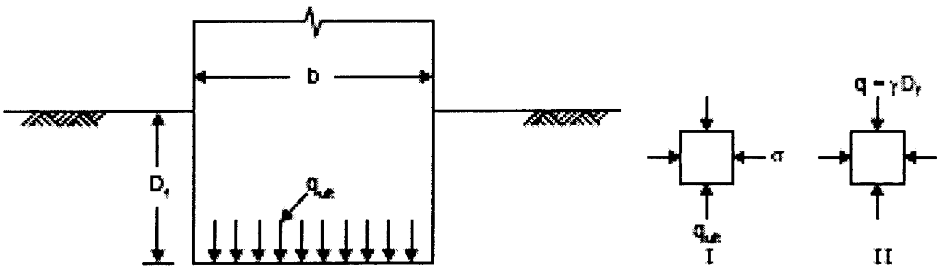
The loaded base sinks into soil like a punch.

The failure surface does not extend up to the ground surface.
No heave is observed. Large vertical strains are involved with practically no lateral deformation.
Failure planes are difficult to locate.

6.4 Bearing Capacity by classical Earth pressure theory of

The classical earth pressure theory assumes that on exceeding a certain stress condition, rupture surfaces are formed in the soil mass. The stress developed upon the formation of the rupture surfaces is treated as the ultimate bearing capacity of the soil.
The bearing capacity may be determined from the relation between the principal stresses at failure. The pertinent methods are those of Rankine, Pauker and Bell.

Rankine's Method
This method, based on Rankine's earth pressure theory, is too approximate and conservative for practical use. However, it is given just as a matter of academic interest.
Rankine uses the relationship between principal stresses at limiting equilibrium conditions of soil elements, one located just beneath the footing and the other just outside it as shown in Fig below.



Rankine's method for bearing capacity of a footing

In element *I*, just beneath the footing, at the base level of the foundation, the applied pressure $q$ult is the major principal stress; under its influence, the soil adjacent to the element tends get pushed out, creating active conditions. The active pressure is σ on the vertical faces to the element. From the relationship between the principal stresses at limiting equilibrium relating to the active state, we have:

$$\sigma = q_{\text{ult}} \cdot K_A = q_{\text{ult}} \left(\frac{1 - \sin \phi}{1 + \sin \phi} \right)$$

In element *II*, just outside the footing, at the base level of the foundation, the tendency of the soil adjacent to the element is to compress, creating passive conditions. The pressure σ on the vertical faces of the element will thus be the passive resistance. This will thus be the major principal stress and the corresponding minor principal stress is $q(= \gamma Df)$, the vertical stress caused by the weight of a soil column on it, or the surcharge dut to the depth of the foundation. From the relationship between the principal stresses at limiting equilibrium relating to the passive state, we have,

$$\sigma = q \cdot K_p = \gamma D_f \cdot K_p = \gamma D_f \left(\frac{1 + \sin \phi}{1 - \sin \phi} \right)$$

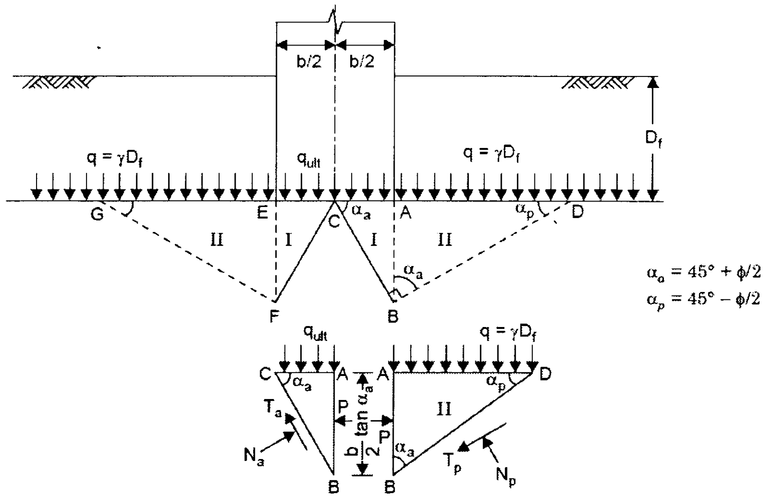
The two values of σ may be equated to get a relationship for q_{ult}:

$$q_{\text{ult}} = \gamma D_f \left(\frac{1 + \sin \phi}{1 - \sin \phi} \right)^2 \quad \text{or,} \quad D_f = \frac{q}{\gamma} \left(\frac{1 - \sin \phi}{1 + \sin \phi} \right)^2$$

This gives the bearing capacity of the footing. It does not appear to take into account the size of the footing. Further the bearing capacity reduces to zero for $D_f = 0$ or for a footing founded at the surface. This is contrary to facts.

An alternative approach based on Rankine's earth pressure theory which takes into account the size *b* of the footing is as follows:

It is assumed that rupture in the soil takes place along *CBD* and *CFG* symmetrically. The failure zones are made of two wedges as shown. It is sufficient to consider the equilibrium of one half.
Wedge *I* is Rankine's active wedge, pushed downwards by $q$ult on *CA*; consequently the vertical face *AB* will be pushed outward.
Wedge *II* is Rankine's passive wedge. The pressure *P* on face *AB* of wedge *I* will be the same as that which acts on face *AB* of wedge *II*; consequently, the soil wedge *II* is pushed up. The surcharge, $q = \gamma D_f$, due to the depth of footing resists this.



Rankine's method taking into account the size of the footing

From wedge *II*,

$$\overline{AB} = \frac{b}{2}\tan\alpha_a = \frac{b}{2}\tan(45° + \phi/2) = \frac{b}{2}\sqrt{N_\phi}$$

$$P = \frac{1}{2}\cdot\gamma\frac{b^2}{4}\cdot N_\phi^2 + \gamma D_f\cdot\frac{b}{2}N_\phi^{3/2}$$

from Rankine's theory for the case with surcharge. From Wedge *I*, similarly,

$$P = \frac{1}{2}\cdot\gamma\cdot\frac{b^2}{4}\cdot\frac{N_\phi}{N_\phi} + q_{ult}\cdot\frac{b}{2}\sqrt{N_\phi}\cdot\frac{1}{N_\phi} = \frac{1}{2}\gamma\cdot\frac{b^2}{4} + q_{ult}\cdot\frac{b}{2}\cdot\frac{1}{\sqrt{N_\phi}}$$

Equating the two values of *P*, we get

$$q_{ult} = \frac{1}{2}\cdot\gamma\cdot\frac{b}{2}\sqrt{N_\phi}\,(N_\phi^2 - 1) + \gamma D_f N_\phi^2$$

This is written as $q_{ult} = \frac{1}{2}\gamma\cdot bN_\gamma + \gamma D_f N_q^2$

where $N_\gamma = \frac{1}{2}\sqrt{N_\phi}\,(N_\phi^2 - 1)$

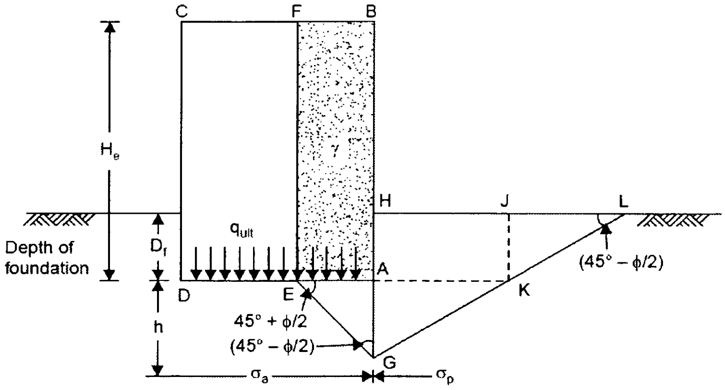
and $N_q = N_\phi^2$,

Both are known as "bearing capacity factors".

6.5 Pauker and Bell's bearing capacity theory of failure

Pauker's Theory

Colonel Pauker, a Russian military engineer, is credited to have derived one of the oldest formulae for the bearing capacity of a foundation in cohesionless soil and the minimum depth of foundation. He was supposed to have used his formula in the 1850's during the construction of fortifications and sea-batteries for the Czarist Naval base of Kronstadt (Pauker, 1889— reported by Jumikis, 1962). His theory was once very popular and was extensively used in Czarist Russia, before the revolution. The theory is set out below as (Figure):



Pauker's method of determination of bearing capacity

Pauker considered the equilibrium of a point say, *G*, in the soil mass underneath the base of the footing, as shown, at a depth *h* below the base, the depth of foundation being D_f below the ground surface. The strip foundation is assumed to transmit a pressure of q_{ult} to the soil at its base.

The classical earth pressure theory for an ideal soil is used under the following assumptions:

(*i*) The soil is cohesionless.

(*ii*) The contact pressure, q_{ult}, is replaced by an equivalent height, H_e, of soil of unit weight, γ, the same as that of the foundation soil:

$$H_e = \frac{q_{ult}}{\gamma}$$

(*iii*) At imminent failure, it is assumed that a part *AEFB*, obtained by drawing *GE* at (45° – ϕ/2) with respect to *GA* (*G* being chosen vertically below *A*), tears off from the rest of the soil mass.

(*iv*) Under the influence of the weight of the equivalent layer of height *He*, the soil to the left of the vertical section *GA* tends to be pushed out, inducing active earth pressure on *GA*.

(*v*) The soil to be right of *GA* tends to get compressed, thus offering passive earth resistance against the active pressure.

(*vi*) The equilibrium condition at *G* is determined by that of soil prisms *GEA* and *GHJK*. The friction of the soil on the imaginary vertical section, *GA*, is ignored. In other words, the earth pressures act normal to *GA*, *i.e.*, horizontally.

(*vii*) If sliding of soil from underneath the footing is to be avoided, the condition stated by Pauker is $\sigma_p \geq \sigma_a$

By Rankine's earth pressure theory,

$$\sigma_p = \gamma\,(Df + h)\tan^2(45° + \phi/2)$$
$$\sigma_a = \gamma\,(He + h)\tan^2(45° - \phi/2)$$

where ϕ is the angle of internal friction of the soil. And from above relationship the equations reduces to

$$\frac{(D_f + h)}{(H_e + h)} \geq \tan^4(45° - \phi/2)$$

[by dividing by $(H_e + h)\tan^2(45° + \phi/2)$ and noting that

$$\frac{\tan^2(45° - \phi/2)}{\tan^2(45° + \phi/2)} = \tan^4(45° - \phi/2).]$$

The most dangerous point *G* is that for which $\frac{(D_f + h)}{(H_e + h)}$ is a minimum.

By inspection, one can see that this is minimum when $h = 0$; that is to say, the critical point is *A* itself.

Thus,

$$\frac{D_f}{H_e} \geq \tan^4 (45° - \phi/2)$$

This is known as Pauker's equation and is written as:

$$D_f = H_e \tan^4(45° - \phi/2)$$

or, noting,

$$H_e = \frac{q_{ult}}{\gamma},\ D_f = \frac{q_{ult}}{\gamma} \cdot \tan^4 (45° - \phi/2)$$

This may be written in the following form also:

$$q_{ult} = \gamma D_f \tan^4 (45° + \phi/2)$$

In the first form it may be used to determine the minimum depth of foundation and in the second, to determine the ultimate bearing capacity.

Bell's Theory

Bell (1915) modified Pauker-Rankine formula to be applicable for cohesive soils; both friction and cohesion are considered in this equation. With reference to Rankine's method (explained above with figure), from, the stresses, on element *I*,

$$\sigma = q_{ult} \tan^2 (45° - \phi/2) - 2c \tan (45° - \phi/2)$$

or

$$\sigma = \frac{q_{ult}}{N_\phi} - \frac{2c}{\sqrt{N_\phi}}$$

with the usual notation, $N_\phi = \tan^2 (45° + \phi/2)$.

This is from the relationship between the principal stresses in the active Rankine state of plastic equilibrium.

From the stresses on element *II*,

$$\sigma = \gamma D_f \tan^2 (45° + \phi/2) + 2c \tan (45° + \phi/2)$$

or

$$\sigma = \gamma D_f N_\phi + 2c \sqrt{N_\phi}$$

Equating the two values of σ for equilibrium, we have:

$$q_{ult} = \gamma D_f \tan^4 (45° + \phi/2) + 2c \tan (45° + \phi/2)[1 + \tan^2 (45° + \phi/2)]$$

or

$$q_{ult} = \gamma D_f N_\phi^2 + 2c\sqrt{N_\phi}\,(1 + N_\phi)$$

This is Bell's equation for the ultimate bearing capacity of a $c - \phi$ soil at a depth D_f. If $c = 0$, this reduces to

$$\sigma = \gamma\, D_f \tan^2 (45° + \phi/2)$$

For pure clay, with $\phi = 0$, Bell's equation reduces to $q_{ult} = \gamma D_f + 4c$

If D_f is also zero, $q_{ult} = 4c$. This value of considered to be too conservative.

The limitation of Bell's equation that the size of the foundation is not considered may be overcome as in the case of Rankine's equation by considering soil wedges instead of elements.

6.6 Prandtl's theory of failure

Prandtl analysed the plastic failure in metals when punched by hard metal punchers (Prandtl, 1920). This analysis has been adapted to soil when loaded to shear failure by a relatively rigid foundation (Prandtl, 1921). The bearing capacity of a long strip footing on the ground surface may be determined by this theory, illustrated in figure below.

The assumptions in Prandtl's theory are:

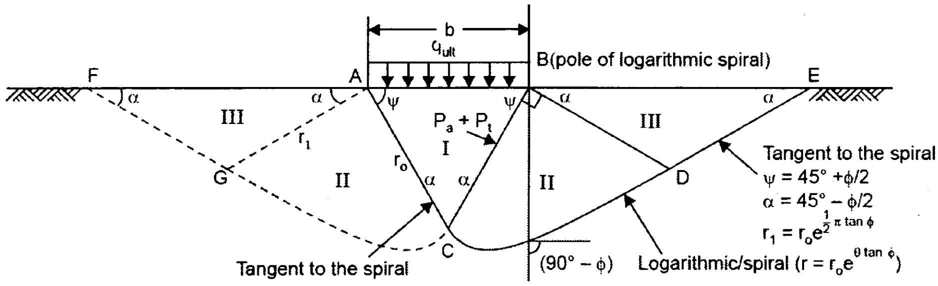
(*i*) The soil is homogeneous, isotropic and weightless.

(*ii*) The Mohr-Coulomb equation for failure envelope $\tau = c + \sigma \tan \phi$ is valid for the soil, as shown in Fig. (*b*).

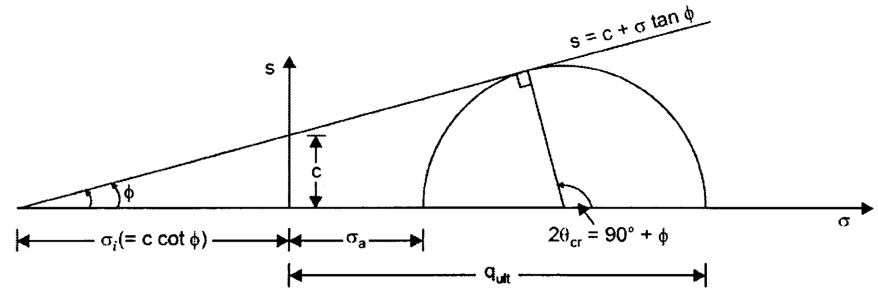
(*iii*) Wedges *I* and *III* act as rigid bodies. The zones in Sectors *II* deform plastically. In the plastic zones all radius vectors or planes through *A* and *B* are failure planes and the curved boundary is a logarithmic spiral.

(*iv*) Wedge *I* is elastically pushed down, tending to push zones *III* upward and outward, which is resisted by the passive resistance of soil in these zones.

(*v*) The stress in the elastic zone *I* is transmitted hydrostatically in all directions.



(a) Prandtl's system



(b) Mohr's circle for active zone

Prandtl's method of determining bearing capacity of a c – ϕ soil

It may be noted that the section is symmetrical up to the point of failure, with an equal chance of failure occurring to either side. (That is why the section to one side, say to the left, is shown by dashed lines). The equilibrium of the plastic sector is considered by Prandtl.

Let *BC* be r_0. The equation to a logarithmic spiral is:

$$r = r_0 \, e^{\theta \tan \phi} \text{ where } \theta \text{ is the spiral angle.}$$

$$\text{Then } BD = r_0 \, e^{(\pi/2) \tan \phi}, \text{ since } \angle CBD = 90° = \pi/2 \text{ rad.}$$

From the Mohr's circle for $c - \phi$ soil, Fig. (*b*), the normal stress corresponding to the cohesion intercept is:

$$\sigma_i = c \cot \phi$$

This is termed the 'initial stress', which acts normally to *BC* in view of assumption (*v*); also q_{ult}, the applied pressure is assumed to be transferred normally on to *BC*. Thus the force on *BC* is

$$(\sigma_i + q_{ult})\,\overrightarrow{BC} \text{ or } r_0(\sigma_i + q_{ult})$$

Moment, M_0, of this force about B is

$$r_0\,(\sigma_i + q_{ult}) \times \frac{r_0}{2}$$

Substituting for σ_i,

$$M_0 = \frac{r_0^2}{2}\,(c \cot \phi + q_{ult}), \text{ counterclockwise}$$

The passive resistance P_p on the face BD is given by

$$P_p = \sigma_i \cdot N_\phi \cdot \overrightarrow{BD}$$

where $N_\phi = \tan^2(45° + \phi/2) = \dfrac{1 + \sin\phi}{1 - \sin\phi}$

This is because σ_i, due to cohesion alone is transmitted by the wedge *BDE*. Its moment about *B*, M_r, is,

$$M_r = P_p \cdot \frac{\overrightarrow{BD}}{2} = \sigma_i N_\phi \cdot \frac{(\overrightarrow{BD})^2}{2} = \cot\phi \cdot N_\phi \cdot \frac{1}{2}\, r_0^2\, e^{\pi \tan\phi}$$

For equilibrium of the plastic zone, equating M_0 and M_r, and rearranging,

$$q_{ult} = c \cot\phi\,(N_\phi \cdot e^{\pi \tan\phi} - 1)$$

This is Prandtl's expression for ultimate bearing capacity of a $c - \phi$ soil. Apparently this leads one to the conclusion that if $c = 0$, $q_{ult} = 0$. This is ridiculous since it is well known that even cohesionless soils have bearing capcity. This anomaly arises chiefly owing to the assumption that the soil is weightless. This was later rectified by Terzaghi and Taylor.

For purely cohesive soils, $\phi = 0$ and the logarithmic spiral becomes a circle and Prandtl's analysis for this special case leads to an indeterminate quantity. But, by applying L' Hospital's rule, for taking limit one finds that

$$q_{ult} = (\pi + 2)c = 5.14c$$

Discussion of Prandtl's Theory

(*i*) Prandtl's theory is based on an assumed compound rupture surface, consisting of an arc of a logarithmic spiral and tangents to the spiral.

(*ii*) It is developed for a smooth and long strip footing, resting on the ground surface.

(*iii*) Prandtl's compound rupture surface corresponds fairly well with the mode of failure along curvilinear rupture surfaces observed from experiments. In fact, for $\phi = 0°$, Prandtl's rupture surface agrees very closely with Fellenius' rupture surface (Taylor, 1948).

(*iv*) Although the theory is developed for a $c - \phi$ soil, the original Prandtl expression for bearing capacity reduces to zero when $c = 0$, contradicting common observations in reality. This anomaly arises from the fact that the weight of the soil wedge directly beneath the base of the footing is ignored in Prandtl's analysis.

This anomaly is sought to be rectified by the Terzagthi/Taylor correction.

(*v*) For a purely cohesive soil, $\phi = 0$, and Prandtl's equation, at first glance, leads to an indeterminate quantity; however this difficulty is overcome by the mathematical technique of evaluating a limit under such circumstances.

Then, for $\phi = 0$, $q_{ult} = (2 + \pi)c = 5.14\,c$

(*vi*) Prandtl's expression, as originally derived, does not include the size of the footing.

6.7 Terzaghi's method of determining bearing capacity of soil

Terzaghi's method is, in fact, an extension and improved modification of Pandtl's (Terzaghi, 1943). Terzaghi considered the base of the footing to be rough, which is nearer facts, and that it is located at a depth *Df* below the ground surface ($Df \leq b$, where b is the width of the footing).

Assumption for Terzaghi's theory:

- ❖The foundation is considered to be shallow if $(D_f \leq B)$, in recent studies the foundation is considered to be shallow if $(D_f / B \leq 4)$. Otherwise it is considered to be deep foundation.
- ❖Foundation is considered to be strip if $(B / L \to 0.00)$.
- ❖The soil from ground surface to the bottom of the foundation is replaced by stress $q = \gamma D_f$
- ❖Soil is homogeneous and Isotropic.
- ❖ The shear strength of soil is represented by Mohr Coulombs Criteria.
- ❖The footing is of strip footing type with rough base. It is essentially a two dimensional plane strain problem.
- ❖Elastic zone has straight boundaries inclined at an angle equal to ϕ to the horizontal.
- ❖Failure zone is not extended above, beyond the base of the footing. Shear resistance of soil above the base of footing is neglected.
- ❖Method of superposition is valid.
- ❖Passive pressure force has three components (P_{PC} produced by cohesion, P_{Pq} produced by surcharge and $P_{p\gamma}$ produced by weight of shear zone).
- ❖Effect of water table is neglected.
- ❖Footing carries concentric and vertical loads.
- ❖Footing and ground are horizontal.
- ❖Limit equilibrium is reached simultaneously at all points. Complete shear failure is mobilized at all points at the same time.
- ❖The properties of foundation soil do not change during the shear failure

Limitations:

- The theory is applicable to shallow foundations.
- As the soil compresses, ϕ increases which is not considered. Hence fully plastic zone may not develop at the assumed ϕ .
- All points need not experience limit equilibrium condition at different loads.
- Method of superstition is not acceptable in plastic conditions as the ground is near failure zone.

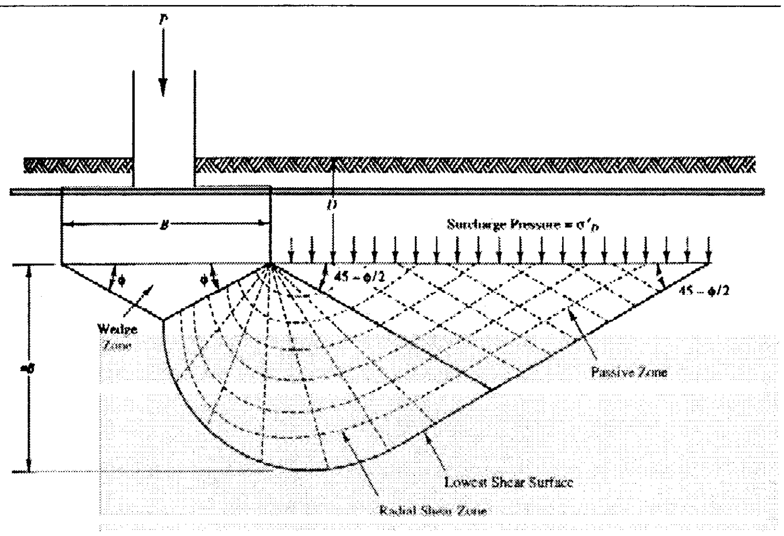
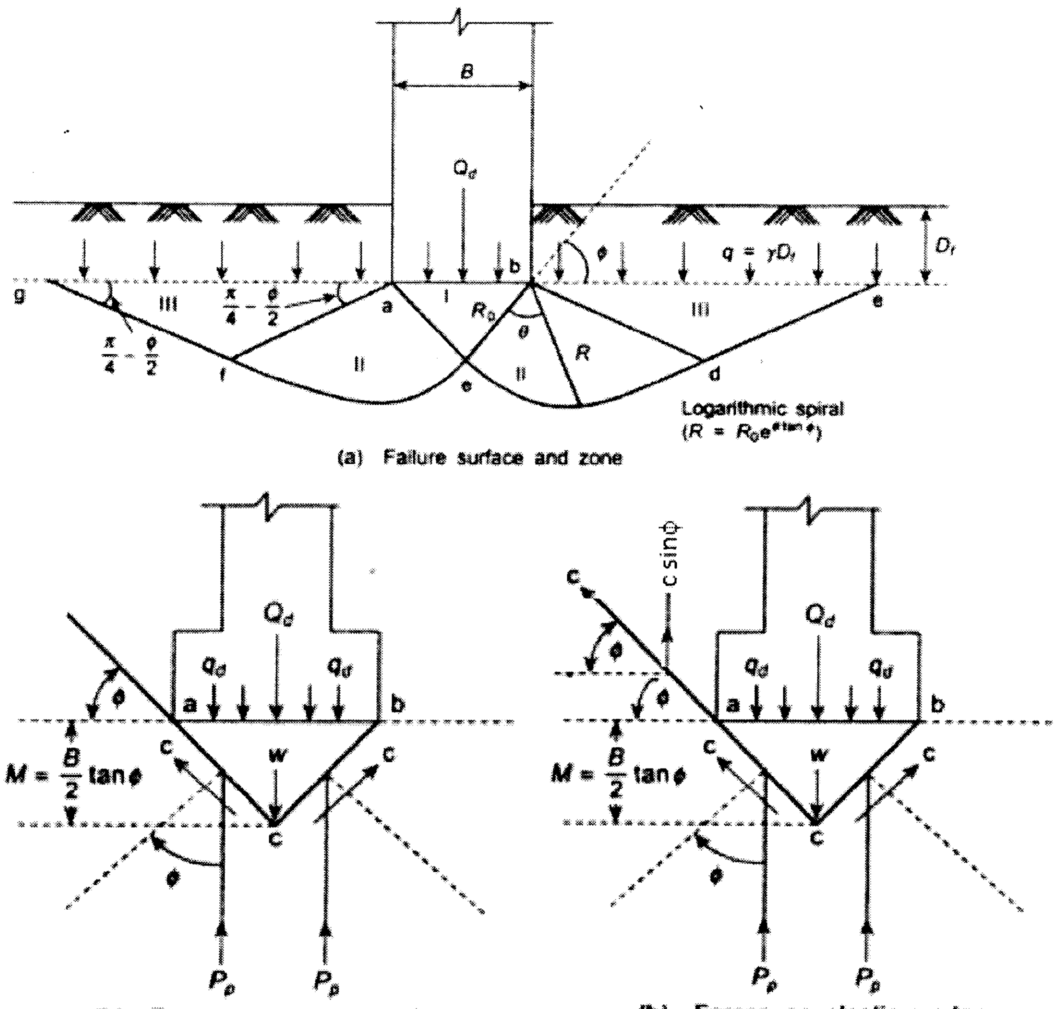


Fig : Terzaghi's concept of Footing with five distinct failure zones in foundation soil.

A strip footing of width B gradually compresses the foundation soil underneathdue to the vertical load from superstructure. Let qf be the final load at which thefoundation soil experiences failure due to the mobilization of plastic equilibrium.The foundation soil fails along the composite failure surface and the region is divided in to five zones, Zone 1 which is elastic, two numbers of Zone 2 which are the zones of radial shear and two zones of Zone 3 which are the zones oflinear shear. Considering horizontal force equilibrium and incorporatingempirical relation,the equation for ultimate bearing capacity is obtained as follows.



(a) Failure surface and zone

(b) Forces on elastic wedge

(b) Forces on elastic wedge

The failure zones do not extend above the horizontal plane passing through base of footing

- ❖The failure occurs when the down ward pressure exerted by loads on the soil adjoining the inclined surfaces on soil wedge is equal to upward pressure.
- ❖Downward forces are due to the load (=$q_u \times$ B) & the weight of soil wedge (=½ x γ x B x B/2 tanØ = ¼ γ B^2 tanø') [$q_u = q_d$]
- ❖Upward forces are the vertical components of resultant passive pressure (Pp) & the cohesion (c') acting along the both inclined surfaces. The vertical component of c' will be c'sinϕ'.

For equilibrium:

Σ Fv = 0

$$\frac{1}{4}\gamma B^2 \tan\phi + q_u \times B = 2P_p + 2c' \times L_i \sin\phi'$$

where Li = length of inclined surface **cb** or **ca**

(**cb**=**ca** = B/2 cosø')

Therefore,

$q_u \times B = 2Pp + Bc' \tan\phi' - \frac{1}{4} \gamma B^2 \tan\phi'$ ....(i)

The resultant passive pressure (Pp) on the surface cb & ca constitutes three components i.e. $(Pp)_r$, $(Pp)_c$ & $(Pp)_q$,

Thus,

$Pp = (Pp)_r + (Pp)_c + (Pp)_q$....................................ii

Then equation (i) becomes

$q_u \times B = 2[(Pp)_r + (Pp)_c + (Pp)_q] + Bc' \tan\phi' - \frac{1}{4} \gamma B^2 \tan\phi'$

Substituting; $2 (Pp)r - \frac{1}{4} rB^2 \tan\phi^1 = B \times \frac{1}{2} \gamma BNr$ iii

$2 (Pp)q = B \times \gamma D Nq$ iv

& $2 (Pp)c + Bc^1 \tan\phi^1 = B \times c' Nc$; v

We get,

$$\mathbf{qu = c' Nc + \gamma\, Df\, Nq + 0.5\, \gamma\, B\, N\gamma} \text{........................vi}$$

This is Terzaghi's Bearing capacity equation for determining ultimate bearing capacity of strip footing. Where Nc, Nq & Nr are Terzaghi's bearing capacity factors & depends on angle of shearing resistance (ø).

6.8 Effect of water table on bearing Capacity

The basic theory of bearing capacity is derived by assuming the water table tobe at great depth below and not interfering with the foundation. However, the presence of water table at foundation depth affects the strength of soil. Further, the unit weight of soil to be considered in the presence of water table is submerged density and not dry density. Hence, the reduction coefficients $R_{W1}$and R_{W2} are used in second and third terms of bearing capacity equation toconsider the effects of water table.

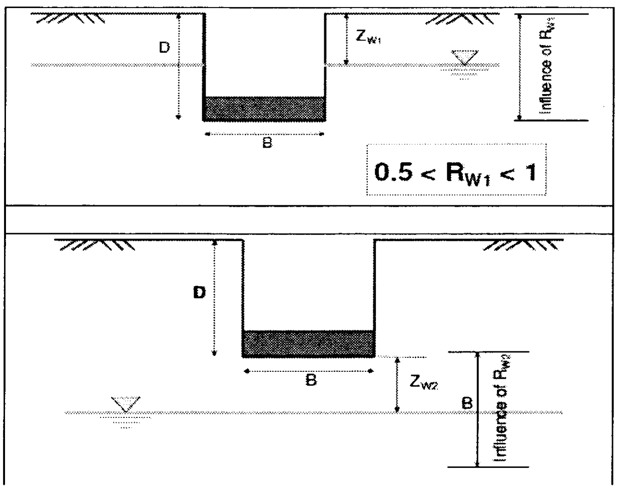


Fig : Effect of water table on bearing capacity

Ultimate bearing capacity with the effect of water table is given by,

$$q_f = cN_c + \gamma DN_q R_{w1} + 0.5\gamma BN_\gamma R_{w2}$$

Here, $R_{w1} = \frac{1}{2}\left[1 + \frac{Z_{w1}}{D}\right]$

where Z_{W1} is the depth of water table from ground level.

1. $0.5 < R_{w1} < 1$
2. When water table is at the ground level ($Z_{w1} = 0$), $R_{w1} = 0.5$
3. When water table is at the base of foundation ($Z_{w1} = D$), $R_{w1} = 1$
4. At any other intermediate level, R_{w1} lies between 0.5 and 1

Here, $R_{w2} = \frac{1}{2}\left[1 + \frac{Z_{w2}}{B}\right]$

where Z_{W2} is the depth of water table from foundation level.

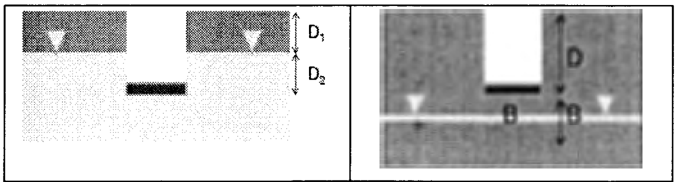
1. $0.5 < R_{w2} < 1$
2. When water table is at the base of foundation ($Z_{w2} = 0$), $R_{w2} = 0.5$
3. When water table is at a depth B and beyond from the base of foundation ($Z_{w2} >= B$), $R_{w2} = 1$
4. At any other intermediate level, R_{w2} lies between 0.5 and 1

Density of soil :

In geotechnical engineering, one deals with several densities such as dry density, bulk density, saturated density and submerged density. There will always be a doubt in the students mind as to which density to use in a particular case. In case of Bearing capacity problems, the following methodology may be adopted.

1. Always use dry density as it does not change with season and it is always smaller than bulk or saturated density.
2. If only one density is specified in the problem, assume it as dry density and use.
3. If the water table correction is to be applied, use saturated density in stead of dry density. On portions above the water table, use dry density.
4. If water table is somewhere in between, use equivalent density as follows. In the case shown in Fig. γ_{eq} should be used for the second term and γ_{sat} for the third term. In the case shown in Fig. γ_d should be used for second term and γ_{eq} for the third term

$$\gamma_{eq} = \frac{\gamma_1 D_1 + \gamma_2 D_2}{D_1 + D_2}$$



(a) Water table above base | (b)Water table below base

Figure: Evaluation of equivalent density

6.9 . Extension of Terzaghi's bearing capacity Theory

For General shear failure:

Type of foundation	Ultimate bearing capacity q_u
Strip Footing	$q_u = cN_c + qN_q + \frac{1}{2}\gamma BN_\gamma$
Square footing	$q_u = 1.3cN_c + qN_q + 0.4\gamma BN_\gamma$
Circular footing	$q_u = 1.3cN_c + qN_q + 0.3\gamma BN_\gamma$

C : Cohesive.

$q = \gamma D_f$

B: Foundation width (Diameter if circular).

N_c, N_q, N_γ : Bearing capacity factors given from table as function of angle of friction ϕ.

Rectangle footing:

$$q_f = (1 + 0.3\frac{B}{L})cN_c + \gamma DN_q + (1 - 0.2\frac{B}{L})0.5\gamma BN_\gamma$$

For Local shear failure:

Type of foundation	Ultimate bearing capacity q_u
Strip Footing	$q_u = \frac{2}{3}cN'_c + qN'_q + \frac{1}{2}\gamma BN'_\gamma$
Square footing	$q_u = 0.867cN'_c + qN'_q + 0.4\gamma BN'_\gamma$
Circular footing	$q_u = 0.867cN'_c + qN'_q + 0.3\gamma BN'_\gamma$

N'_c, N'_q, N'_γ : Factors for bearing capacity given from table or from table but replace Φ' by Φ :

$$\Phi' = \tan^{-1}\left(\frac{2}{3}\tan\Phi\right)$$

6.10. Recent bearing capacity Theory

Meyerhof's equations (General bearing capacity equation):

Terzagi equations neglect:

✓Rectangular footings.

✓Inclination of loads.

✓Shear strength of soil above the foundation.

Meyerhof's equation takes in consideration these variables:

$$q_u = c'\lambda_{cs}\lambda_{cd}\lambda_{ci}N_c + q\lambda_{qs}\lambda_{qd}\lambda_{qi}N_q + \tfrac{1}{2}\lambda_{\gamma s}\lambda_{\gamma d}\lambda_{\gamma i}\gamma BN_\gamma$$

where λ_{cs}, λ_{qs}, and $\lambda_{\gamma s}$ = shape factors

λ_{cd}, λ_{qd}, and $\lambda_{\gamma d}$ = depth factors

λ_{ci}, λ_{qi}, and $\lambda_{\gamma i}$ = inclination factors

N_c, N_q, N_γ : (From bearing capacity factor of Meyerhof's Table)

Skempton's equation for clay without inclination:

$$q_u = 5c\left(1 + 0.2\frac{D_f}{B}\right)\left(1 + 0.2\frac{B}{L}\right)$$

Vesic's equation (Consider compressibility of soil):

$$q_u = cN_c F_{cs} F_{cd} F_{cc} + qN_q F_{qs} F_{qd} F_{qc} + 0.5\gamma B N_\gamma F_{\gamma s} F_{\gamma d} F_{\gamma c}$$

N_c, N_q, N_γ: Table

$F_{cc}, F_{qc}, F_{\gamma c} \Rightarrow$ Soil Compressibility factors.

Brinch Hansen's Bearing Capacity equation:

As mentioned in previous section, bearing capacity depends on many factors and Terzaghi's bearing capacity equation does not take in to consideration all the factors. Brinch Hansen and several other researchers have provided a comprehensive equation for the determination bearing capacity called Generalised Bearing Capacity equation considering the almost all the factors mentioned above. The equation for ultimate bearing capacity is as follows from the comprehensive theory

$$q_f = cN_c s_c d_c i_c + qN_q s_q d_q i_q - 0.5\gamma B N_\gamma s_\gamma d_\gamma i_\gamma$$

Here, the bearing capacity factors are given by the following expressions which depend on ϕ.

$$N_c = (N_q - 1)\cot\phi$$

$$N_q = (e^{\pi\tan\phi})\tan^2(45 + \frac{\phi}{2})$$

$$N_\gamma = 1.5(N_q - 1)\tan\phi$$

Equations are available for shape factors (s_c, s_q, s_γ), depth factors (d_c, d_q, d_γ) and load inclination factors (i_c, i_q, i_γ). The effects of these factors is to reduce the bearing capacity.

Factors	Meyerhof	Hansen	Vesic
s_c	$1+0.2N_\phi \frac{B}{L}$	$1+\frac{N_q}{N_c}\frac{B}{L}$	The shape and depth factors of Vesic are the same as those of Hansen.
s_q	$1+0.1N_\phi \frac{B}{L}$ for $\phi>10°$	$1+\frac{B}{L}\tan\phi$	
s_γ	$s_\gamma = s_q$ for $\phi>10°$ $s_\gamma = s_q = 1$ for $\phi=0$	$1-0.4\frac{B}{L}$	
d_c	$1+0.2\sqrt{N_\phi}\frac{D_f}{B}$	$1+0.4\frac{D_f}{B}$	
d_q	$1+0.1\sqrt{N_\phi}\frac{D_f}{B}$ for $\phi>10°$	$1+2\tan\phi(1-\sin\phi)^2\frac{D_f}{B}$	
d_γ	$d_\gamma = d_q$ for $\phi>10°$ $d_\gamma = d_q = 1$ for $\phi=0$	1 for all ϕ Note; Vesic's s and d factors = Hansen's s and d factors	
i_c	$\left(1-\frac{\alpha°}{90}\right)^2$ for any ϕ	$i_q-\frac{1-i_q}{N_q-1}$ for $\phi>0$	Same as Hansen for $\phi>0$
		$0.5\left(1-\frac{Q_h}{A_f c_a}\right)^{\frac{1}{2}}$ for $\phi=0$	$1-\frac{mQ_h}{A_f c_a N_c}$
i_q	$i_q = i_c$ for any ϕ	$\left(1-\frac{0.5Q_h}{Q_u+A_f c_a\cot\phi}\right)^5$	$\left(1-\frac{Q_h}{Q_u+A_f c_a\cot\phi}\right)^m$
i_γ	$\left(1-\frac{\alpha°}{\phi°}\right)^2$ for $\phi>0$ $i_\gamma=0$ for $\phi=0$	$\left(1-\frac{0.7Q_h}{Q_u+A_f c_a\cot\phi}\right)^5$	$\left(1-\frac{Q_h}{Q_u+A_f c_a\cot\phi}\right)^{m+1}$

$N_\gamma = (N_q-1)\tan(1.4\phi)$ (Meyerhof)

$N_\gamma = 1.5(N_q-1)\tan\phi$ (Hansen)

$N_\gamma = 2(N_q+1)\tan\phi$ (Vesic)

6.11 Bearing capacity from in- situ tests (Plate load Test)

Field Tests are performed in the field. You have understood the advantages offield tests over laboratory tests for obtaining the desired property of soil. Thebiggest advantages are that there is no need to extract soil sample and theconditions during testing are identical to the actual situation.

Major advantages of field tests are

· Sampling not required

· Soil disturbance minimum

Major disadvantages of field tests are

· Laborious

· Time consuming

· Heavy equipment to be carried to field

· Short duration behavior

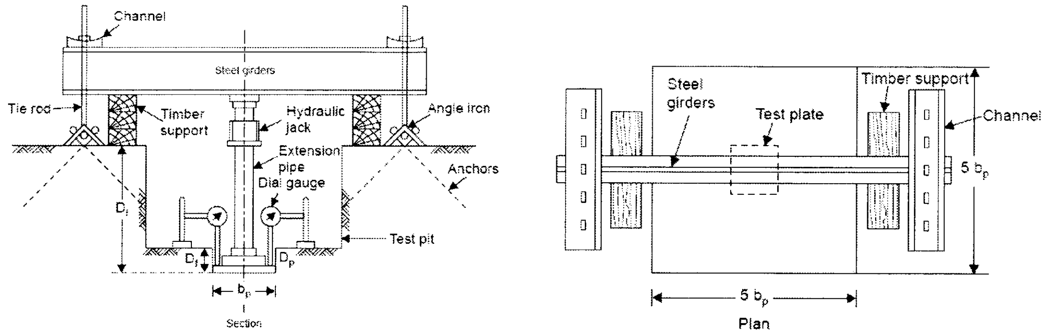


Figure: Typical set up for Plate Load test assembly

1. It is a field test for the determination of bearing capacity and settlement characteristics of ground in field at the foundation level.
2. The test involves preparing a test pit up to the desired foundation level.
3. A rigid steel plate, round or square in shape, 300 mm to 750 mm in size, 25 mm thick acts as model footing.
4. Dial gauges, at least 2, of required accuracy (0.002 mm) are placed on plate on plate at corners to measure the vertical deflection.
5. Loading is provided either as gravity loading or as reaction loading. For smaller loads gravity loading is acceptable where sand bags apply the load.
6. In reaction loading, a reaction truss or beam is anchored to the ground. A hydraulic jack applies the reaction load.
7. At every applied load, the plate settles gradually. The dial gauge readings are recorded after the settlement reduces to least count of gauge (0.002 mm) & average settlement of 2 or more gauges is recorded.
8. Load Vs settlement graph is plotted as shown. Load (P) is plotted on the horizontal scale and settlement (s) is plotted on the vertical scale.
9. Red curve indicates the general shear failure & the blue one indicates the local or punching shear failure.
10. The maximum load at which the shear failure occurs gives the ultimate bearing capacity of soil.

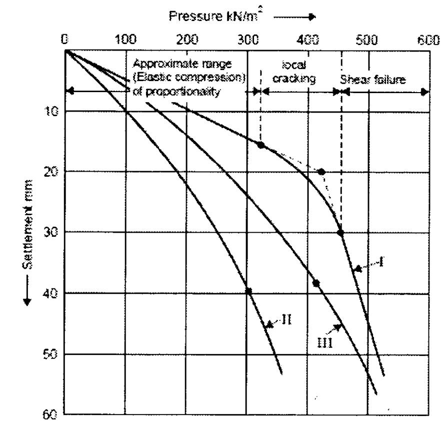
Reference can be made to IS 1888 - 1982.

The failure point is obtained as the point corresponding to the intersection of the initial and final tangents.

The value of q_{ult} here is given by $\frac{1}{2}\gamma b_p N_\gamma$.

$$\frac{S}{S_p} = \left[\frac{b(b_p + 0.3)}{b_p(b + 0.3)}\right]^2$$

where S = settlement of the proposed foundation (mm),
S_p = settlement of the test plate (mm), (same units)
b = size of the proposed foundation (m), and
b_p = size of the test plate (m).
This is applicable for sands.



However, the relationship is simpler for clays, since the modulus value E_s, for clays is reasonably constant:

$$\frac{S}{S_p} = \frac{b}{b_p}$$

Equation may be put in a slightly simplified form as follows:

$$S = S_p \left[\frac{2b}{b + 0.3} \right]^2$$

where S_p = Settlement of a test plate of 300 mm square size,
and S = Settlement of a footing of width b.

The advantages of Plate Load Test are

1. It provides the allowable bearing pressure at the location considering both shear failure and settlement.
2. Being a field test, there is no requirement of extracting soil samples.
3. The loading techniques and other arrangements for field testing are identical to the actual conditions in the field.
4. It is a fast method of estimating ABP(Allowable bearing Presssure) and Pressure– Settlement behaviour of ground.

The disadvantages of Plate Load Test are

1. The test results reflect the behaviour of soil below the plate (for a distance of ~2Bp), not that of actual footing which is generally very large.
2. It is essentially a short duration test. Hence, it does not reflect the long term consolidation settlement of clayey soil.
3. Size effect is pronounced in granular soil. Correction for size effect is essential in such soils.
4. It is a cumbersome procedure to carry equipment, apply huge load and carry out testing for several days in the tough field environment.

Bearing capacity from SPT tests based on settlement:

Bearing capacity of Sand based on settlement, by Bowels

$$q_{net}(\text{kN/m}^2) = \frac{N_{60}}{0.05} F_d \left[\frac{S_e(\text{mm})}{25} \right] \quad (\text{for } B \le 1.22\text{m})$$

$$F_d = \text{depth factor} = 1 + 0.33\left(\frac{D_f}{B} \right) \le 1.33$$

$$q_{net}(\text{kN/m}^2) = \frac{N_{60}}{0.08} \left(\frac{B + 0.3}{B} \right)^2 F_d \left[\frac{S_e(\text{mm})}{25} \right] \quad (\text{for } B > 1.22\text{m})$$

where B = foundation width (m)
S_e = settlement

Some empirical relationships are:

Teng (1969) has proposed the following equation for the graphical relationship of Terzaghi and Peck for a settlement of 25 mm:

$$q_{na} = 34.3\,(N-3)\left(\frac{b+0.3}{2b}\right)^2 R_\gamma \,.\, R_d$$

where q_{na} = net allowable soil pressure in kN/m^2 for a settlement of 25 mm,

N = Standard penetration value corrected for overburden pressure and other applicable factors,

b = width of footing in metres,

R_γ = correction factor for location of water table,

and R_d = Depth factor ($= 1 + D_f/b) \le 2$, where D_f = depth of footing in metres.

The modified equation of Teng is as follows:

$$q_{na} = 51.45(N-3)\left(\frac{b+0.3}{2b}\right)^2 R_\gamma \,.\, R_d$$

The notation is the same

Meyerhof (1956) has proposed slightly different equations for a settlement of 25 mm, but these yield almost the same results as Teng's equation:

$$q_{na} = 12.25\, NR_\gamma \,.\, R_d\,, \text{ for } b \le 1.2 \text{ m}$$

$$q_{na} = 8.17\, N\left(\frac{b+0.3}{b}\right) .\, R_\gamma \,.\, R_d\,, \text{ for } b > 1.2 \text{ m}$$

The notation is the same as those of Eqs. 14.119 and 14.120.

Modified equation of Meyerhof is as follows:

$$q_{na} = 18.36\, NR_\gamma \,.\, R_d\,, \text{ for } b \le 1.2 \text{ m}$$

$$q_{na} = 12.25\, N\left(\frac{b+0.3}{b}\right) R_\gamma \,.\, R_d\,; \text{ for } b > 1.2 \text{ m}$$

The modified equations of Teng and Meyerhof are based on the recommendation of Bowles (1968).

The I.S. code of practice gives for a settlement of 40 mm; but, it does not consider the depth effect.

Teng (1969) also gives the following equations for bearing capacity of sands based on the criterion of shear failure:

$$q_{\text{net ult}} = 1/6[3N^2 .b\, R_\gamma + 5(100 + N^2)D_f \,.R_q]$$

(for continuous footings)

$$q_{\text{net ult}} = 1/6[2N^2\, b\, R_\gamma + 6\,(100 + N^2)D_f \,.\, R_q]$$

(for square or circular footings)

Here again,

$q_{\text{net ult}}$ = net ultimate soil pressure in kN/m^2.,

N = Standard penetration value, after applying the necessary corrections,

b = width of continuous footing (side, if square, and diameter, if circular in metres),

D_f = depth of footing in metres, and

R_γ and R_q = correction factors for the position of the ground water table,

With a factor of safety of 3, the net safe bearing capacity q_{ns}, is given by

$$q_{ns} = \frac{1}{18}\,[3N^2 b\, R_\gamma + 5(100 + N^2)D_f R_q] - \frac{2}{3}\,\gamma \,.\, D_f$$

(for continuous footings)

$$q_{ns} = \frac{1}{18}\,[2N^2 b\, R_\gamma + 6(100 + N^2)D_f R_q] - \frac{2}{3}\,\gamma \,.\, D_f$$

(for square or circular footings)

In some books, Teng's equation is in different form as: (Verify required)

From bearing capacity considerations,
for very long and strip footings:

$$q_{ult-net} = \frac{1}{62}\left[3N^2BR'_w + 5(100 + N^2)DR_w\right]$$

for square and circular footings:

$$q_{ult-net} = \frac{1}{31}\left[N^2BR'_w + 3(100 + N^2)DR_w\right]$$

From settlement considerations the equations for safe bearing pressure are as follows

$$q_{safe-pr} = 3.5(N - 3)\left(\frac{B + 0.3}{2B}\right)^2 R'_w C_D \quad \text{for } S_p = 2.5 \text{ cm}$$

$$= 1.4(N - 3)\left(\frac{B + 0.3}{2B}\right)^2 R'_w C_D S_p$$

for a specified permissible settlement of S_p in cm

6.12 Types of Settlement and their relationship:

1. The downward movement of a building structure due to consolidation of soil beneath the foundation.
2. The sinking of solid particles of aggregate in fresh concrete or mortar after its placement and before its initial set.

Types of settlement (In structure)
a.Angular Distortion
b.Tilt
c. Different settlement

Computation of settlement is not required for light structures and computation settlement is necessary for heavy structures and can be calculated in several methods.

Settlement in soil:

Immediate Settlement: Occurs immediately after the construction. This is computed using elasticity theory (Important for Granular soil)

Primary Consolidation: Due to gradual dissipation of pore pressure induced by external loading and consequently expulsion of water from the soil mass, hence volume change. (Important for Inorganic clays)

Secondary Consolidation: Occurs at constant effective stress with volume change due to rearrangement of particles. (Important for Organic soils)

- Settlement under Loads (Components of settlement):
  - S_e - Elastic or Immediate Settlement
  - S_c - Consolidation Settlement
  - S_s - Secondary Settlement

$S = S_e + S_c + S_s$

- Net elastic settlement of flexible footing

$$S_e = q_n B \frac{(1-\mu^2)}{E_s} I_f$$

S_e = elastic settlement
B = width of foundation,
E_s = modulus of elasticity of soil,
μ = Poisson's ratio,
q_n = net foundation pressure,
I_f = influence factor.

Influence factor as per Bowels

Shape	I_f (average values) Flexible footing	Rigid footing
Circle	0.85	0.88
Square	0.95	0.82
Rectangle	1.20	1.06
L/B = 1.5	1.20	1.06
2.0	1.31	1.20
5.0	1.83	1.70
10.0	2.25	2.10
100.0	2.96	3.40

- Elastic Settlement of Rigid Footing

  $S_{ef} = C_r d_f S_e$

  Where,

  S_{ef} = Final elastic settlement

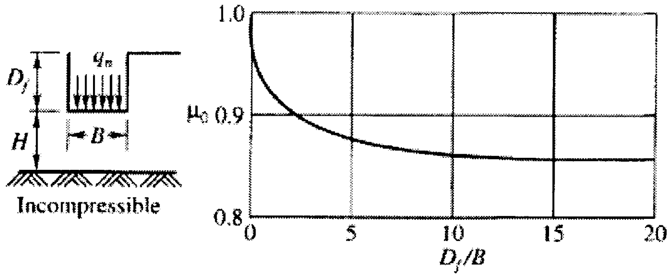
  C_r = Rigidity Factor (Highly rigid footing, Cr = 0.8)

  D_f = Depth factor

  S_e = Settlement for a surface flexible footing

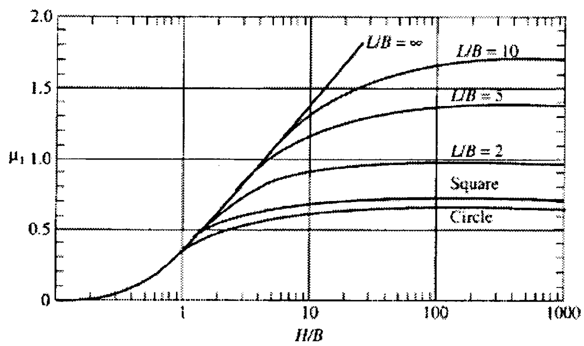
Janbu, Bjerrum and Kjaernsli's method of determining elastic settlement under undrained conditions

$$S_e = \frac{\mu_0 \mu_1 q_n B}{E_s}$$



- Consolidation and secondary Settlement (studied in Soil Mechanics- Refer Arora)

$$S_c = H\frac{C_c}{1+e_0}\log\frac{p_0+\Delta p}{p_0}$$



6.13 Permissible settlement and allowable bearing pressure :

It is amount of vertical displacement in which settlenment of structure falls in permissible limit.The permissible settlement of spread and mat foundation generally taken 25mm and 40mm according to Terzaghi .The allowable bearing capacity or pressure is governed by allowable settlement. It is correspond value of permissible settlement. Computation of allowable bearing pressure is made according to the procedure for doing plate load test and penetration test.

6.14 Steps involved in the proportion of footings:

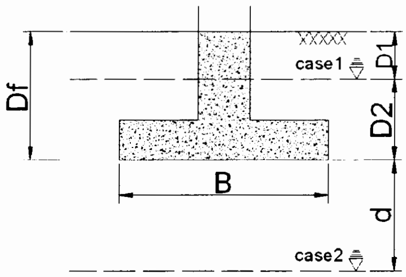
To reduce the differential settlement due to variations of loads proportion of all footings is desirable for uniform settlement.

Steps for proportioning of spread footing – (From Arora Book)

1. Determine dead load including weight of footing.
2. Detrmine the footing subjected to maximum load.
3. Compute the ratio of live load to dead load for each footing .
4. Identify the governing footing having maximum live load to dead load ratio is the governing footing.
5. Find the area of governing footing (Ag).
6. Area of governing Footing
   = DL+LL / Allowable bearing capacity of soil

7. Determine the design service load for all the footing.
8. Determine the design bearing capacity (q_d) of all the footings except the governig footing.
9. Design bearing capacity
   = service load of governing footing $/A_g$
10. Determine area under the other footing (A).
11. A = Service load of that loading $/q_d$

Effect of water table in bearing capacity equations:



Case I) Water table is located at depth D_1 so that $0 \le D_1 \le D_f$:

$q = \gamma D_1 + \gamma' D_2$

$\gamma = \gamma' = \gamma_{sat} - \gamma_w$

Case II) Water table is located at depth d below the foundation so that $0 \le d \le B$:

$q = \gamma D_f$

$\gamma = \bar{\gamma} = \gamma' + \frac{d}{B}(\gamma - \gamma')$

Case III) Water table is located at depth d below the foundation so that $d > B$:
No changes in equations.

Factor of safety:

Ultimate bearing capacity

$q_u \Rightarrow$ Gross ultimate bearing capacity

$((q_u)_{net} = q_u - q) \Rightarrow$ Net ultimate bearing capacity

$q_{all} \Rightarrow$ Gross allowable bearing capacity

$(q_{all})_{net} \Rightarrow$ Net allowable bearing capacity

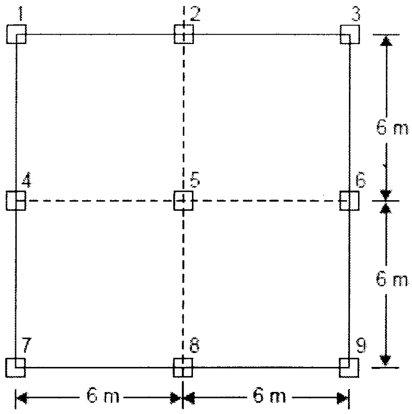
$Q_u \Rightarrow$ *Gross* Ultimate load.

$$\Rightarrow q_{all} = \frac{q_u}{FS}$$

$$\Rightarrow (q_{all})_{net} = \frac{(q_u)_{net}}{FS} = \frac{q_u - q}{FS} = q_{all} - \frac{q}{FS}$$

FS = (3 – 4) for bearing capacity Factor of safety with respect to shear:

Example : A building is supported on nine columns as shown in Fig. below and column loads are indicated. Determine the required areas of the column footings:



Column No.	1	2	3	4	5	6	7	8	9
Dead Load (kN)	180	360	240	300	600	360	180	360	210
Max. Live Load (kN)	180	400	210	300	720	360	120	300	180

At the selected depth of 1.5 m the allowable bearing capacity is 270 kN/m2. $\gamma = 20$ kN/m³.

Solution:

Dead load plus maximum live load, maximum live load to dead load ratio, reduced live load and dead load plus reduced live load are all determined and tabulated for all the columns (A reduction factor of 50% is used for *LL*).

Column No.	1	2	3	4	5	6	7	8	9
Dead Load (kN)	180	360	240	300	600	360	180	360	210
Max. LL (kN)	180	400	210	300	720	360	120	300	180
DL + Max. LL (kN)	360	760	450	600	1320	720	300	660	390
Max. LL/DL	1.00	1.11	0.88	1.00	1.20	1.00	0.67	0.83	0.86
Reduced LL (kN)	90	200	105	150	360	180	60	150	90
DL + Reduced LL (kN)	270	560	345	450	960	540	240	510	300

Column No. 5 has the maximum LL to DL ratio of 1.20 and hence it governs the design.

Assuming the thickness of the footing as 1 m,

allowable soil pressure corrected for the weight of the footing = $(270 - 1 \times 20) = 250$ kN/m²

$\therefore$ Area of footing for column No. 5 = $\dfrac{1320}{250} = 5.28$ m²

Reduced Load for this column = 960 kN

Reduced allowable pressure $= \dfrac{\text{Reduced load}}{\text{Area}} + \text{Weight of footing}$

$= \dfrac{960}{5.28} + 20 = 182 + 20 =$ **200 kN/m²**

The footing sizes will be obtained by dividing the reduced loads, for each column by the corrected reduced allowable pressure of $\dfrac{960}{5.28}$ or 182 kN/m².

The results are tabulated below:

Column No.	1	2	3	4	5	6	7	8	9
Reduced Load (kN)	270	560	345	450	960	540	240	510	300
Corrected reduced soil pressure (kN/m^2)	182	182	182	182	182	182	182	182	182
Required area (m^2)	1.49	3.07	1.90	2.48	5.28	2.97	1.32	2.80	1.65
Size of footing (m^2)	**1.25**	**1.75**	**1.40**	**1.60**	**2.30**	**1.75**	**1.20**	**1.70**	**1.30**

The thickness of the footing may be varied somewhat with loading. This will somewhat alter the reduced allowable pressures for different footings. The areas of the footings will get increased slightly. However, this refinement is ignored in tabulating the sizes of the square footings.

The structural design of the footings may now be made.