AL- ESRAA COLLEGE UNIVERSITY

Building & Construction Technology Engineering

Engineering Mechanics

First year

Resolution and Composition of Forces

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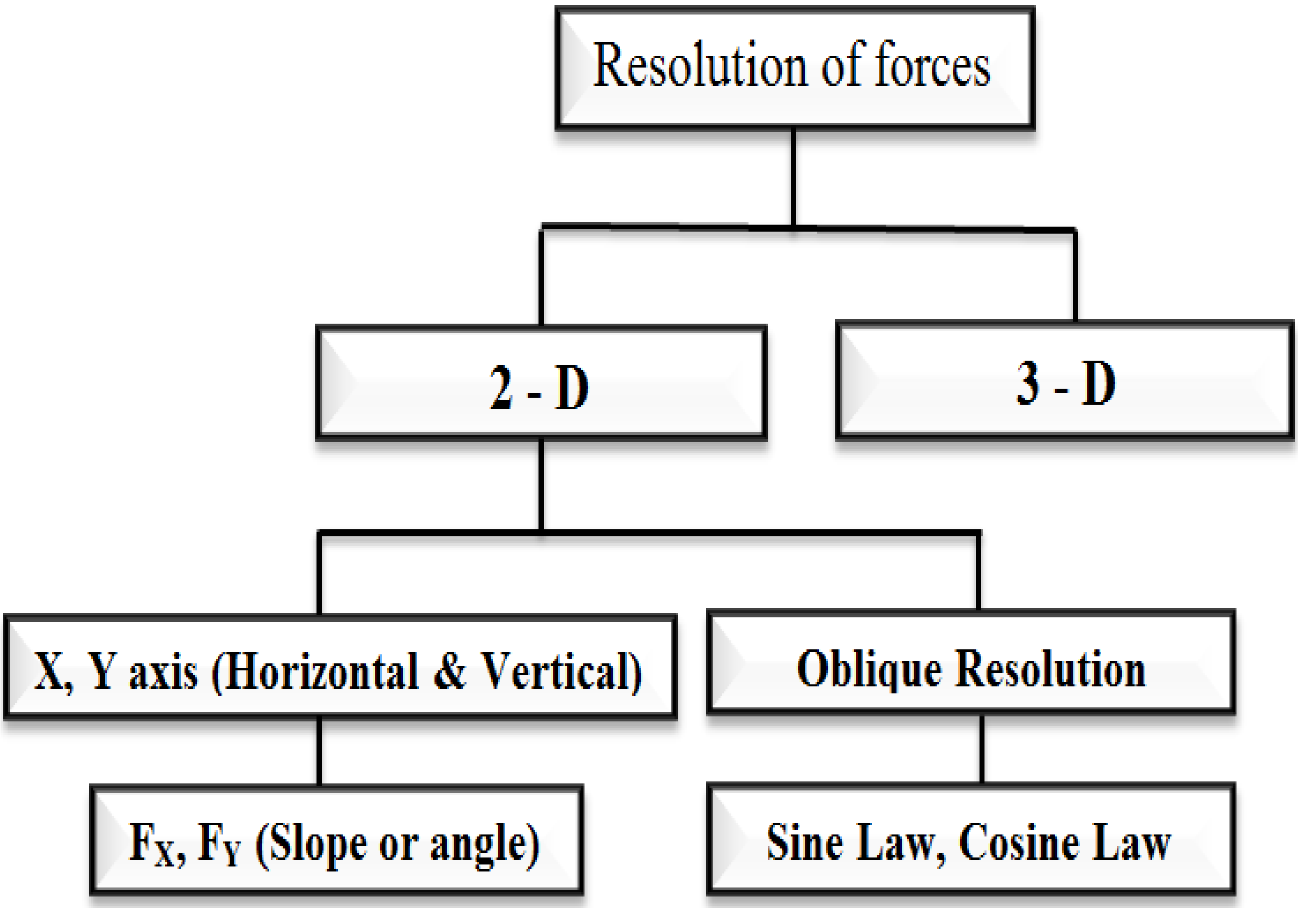
Resolution and Composition of Forces

The replacement of a single force by a several components which will be equivalent in action to the given force is called **Resolution** of a force. The process of replacing a force system by its resultant is called **Composition**. Resolution and Composition determine by Parallelogram and Triangle Law.

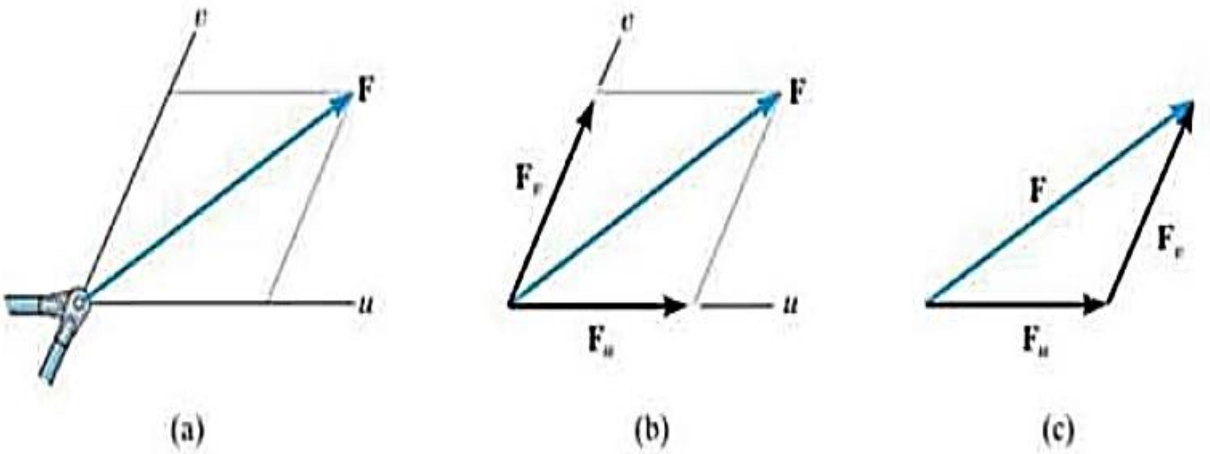
The force is resolved into two rectangular components: vertical component ($\mathbf{F_y}$) and horizontal component ($\mathbf{F_x}$), when resolving a force in to x-y components, we must have information on the direction of the force and the magnitude of the force.



A- Two-Dimensional Resolution of a Force(2-D):

Parallelogram and Triangle Law of Forces

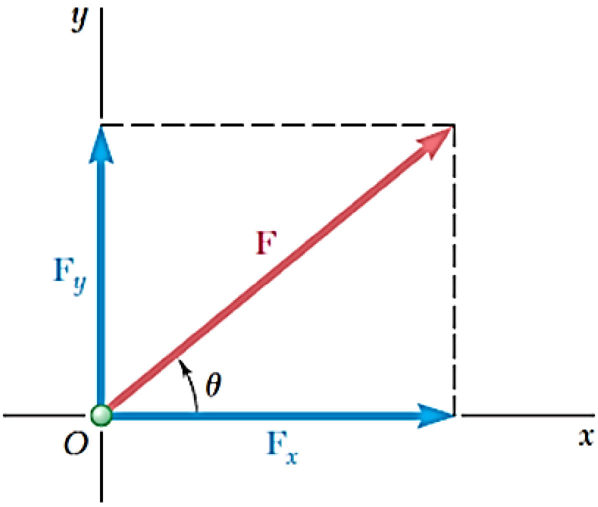
Parallelogram and triangle law use to find the components of a force. For example, in Figure (a) F is to be resolved in to two components along the two members, defined by the *u* and *v* axes. In order to determine the magnitude of each component, a parallelogram is constructed first, by drawing lines starting from the tip of F. one line parallel to *u*, and the other line parallel to *v*. These lines then intersect with the *v* and *u* axes, forming a parallelogram. The force components F*u* and F*v* are then established by simply joining the tail of F to the intersection points on the *u* and *v* axes, Figure (b). This parallelogram can then be reduced to a triangle, which represents the triangle rule, Figure (c). From this, the law of sines can then be applied to determine the unknown magnitudes of the components.



Resolution of a Force along x- and y-directions

When a force is resolved in to two components along the *x* and, y axes, the components are then called rectangular components.

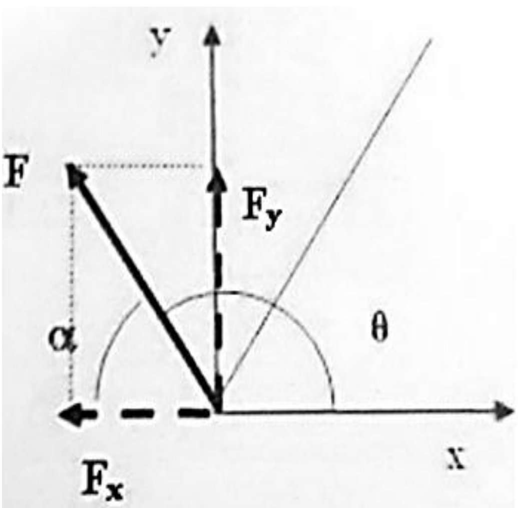
1- When the angle of the force relative to the x- or y-axes (θ) is known.



$$\sin\theta = \frac{F_y}{F} \rightarrow \quad F_y = F\,.\sin\theta$$

$$\cos\theta = \frac{F_x}{F} \rightarrow \quad F_x = F\,.\cos\theta$$

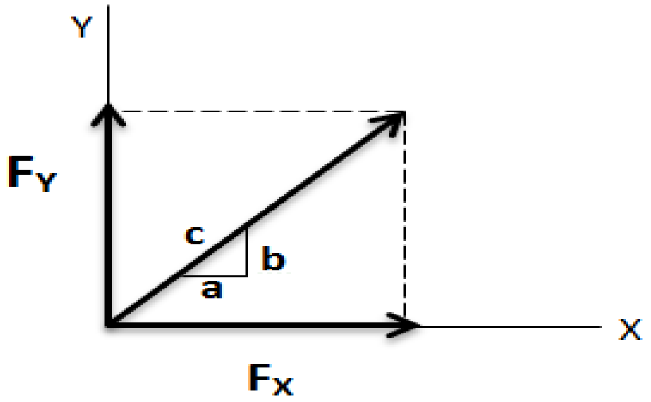
❖ It is usually easiest to find the magnitudes of the components from the acute angle of the triangle defined by the force and the axes. The scalar components can be positive or negative, depending on the quadrant into which **F** points. You need to recognize the signs of the components so they agree with their senses.



$$\sin\theta = \frac{F_y}{F} = \sin(180-\alpha) \rightarrow F_y = F\,.\sin\alpha$$

$$\cos\theta = \frac{F_x}{F} = \cos(180-\alpha) \rightarrow \quad F_x = -\,F\,.\cos\alpha$$

2- When the (small triangle) slop of the force is known.

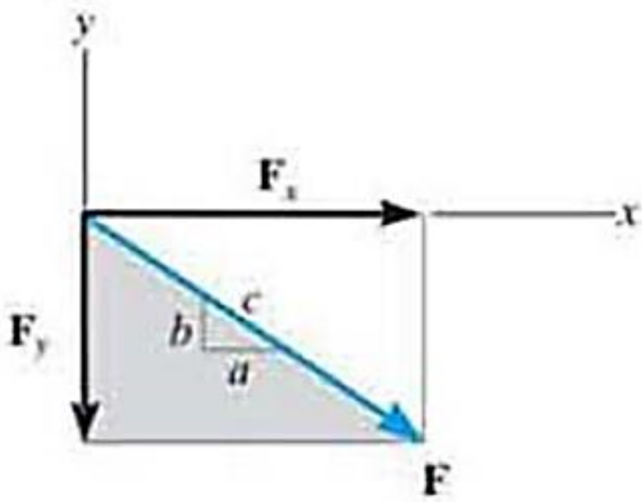


$$c = \sqrt{a^2 + b^2}$$

$$F_y = F \cdot \frac{b}{c}$$

$$F_x = F \cdot \frac{a}{c}$$

❖ The scalar components can be positive or negative, depending on the quadrant into which **F** points.



$$c = \sqrt{a^2 + b^2}$$

$$F_y = -F \cdot \frac{b}{c}$$

$$F_x = F \cdot \frac{a}{c}$$

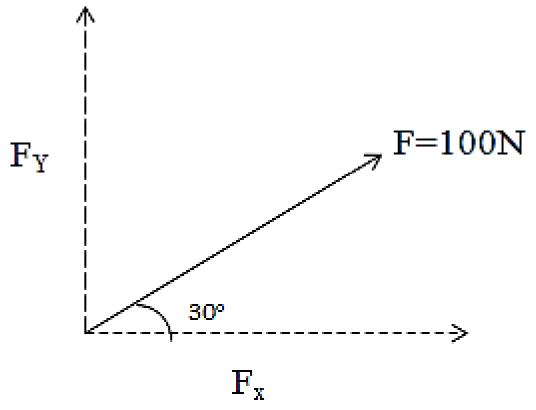
Example: Determine the component in the direction x&y for the (100N) force make angle θ=30° with the positive direction for x.

$$F_x = F \times \cos 30$$

$$F_x = 100 \times \cos 30 = 86.6N \longrightarrow$$

$$F_y = F \times \sin 30$$

$$F_y = 100 \times \sin 30 = 50N \uparrow$$

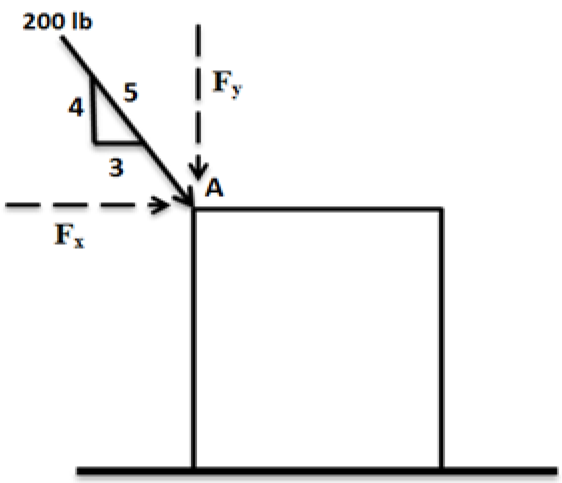


Example: Determine a set of horizontal and vertical components of the 200-lb force shown in figure below.

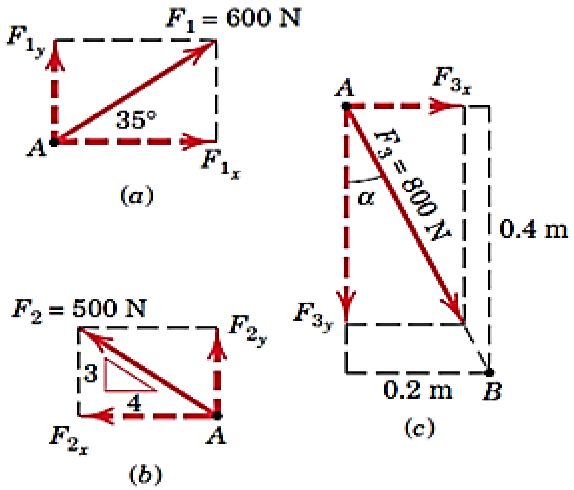
$$r = \sqrt{3^2 + 4^2} = 5$$

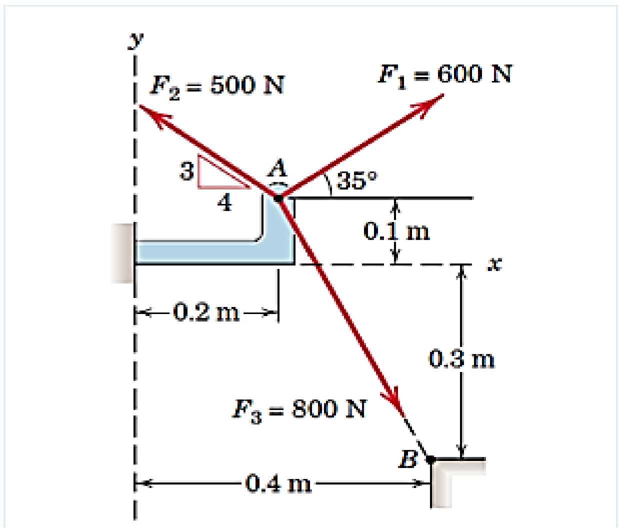
$$F_x = 200 \times \frac{3}{5} = 120 \text{ lb} \longrightarrow$$

$$F_y = 200 \times \frac{4}{5} = 160 \text{ lb} \downarrow$$



<u>Example:</u> The forces F1, F2, and F3, all of which act on point *A* of the bracket, are specified in three different ways. Determine the *x* and *y* scalar components of each of the three forces.





The components of **F**1, from Fig. *a*, are

$$F_{1X} = 600 \times \text{Cos } 35 = 491\text{N} \longrightarrow$$

$$F_{1y} = 600 \times \text{Sin } 35 = 344\text{N} \uparrow$$

The components of **F**2, from Fig. *b*, are

$$F_{2x} = 500 \times \frac{4}{5} = 400 \text{ N} \longleftarrow$$

$$F_{2y} = 500 \times \frac{3}{5} = 300\ N \uparrow$$

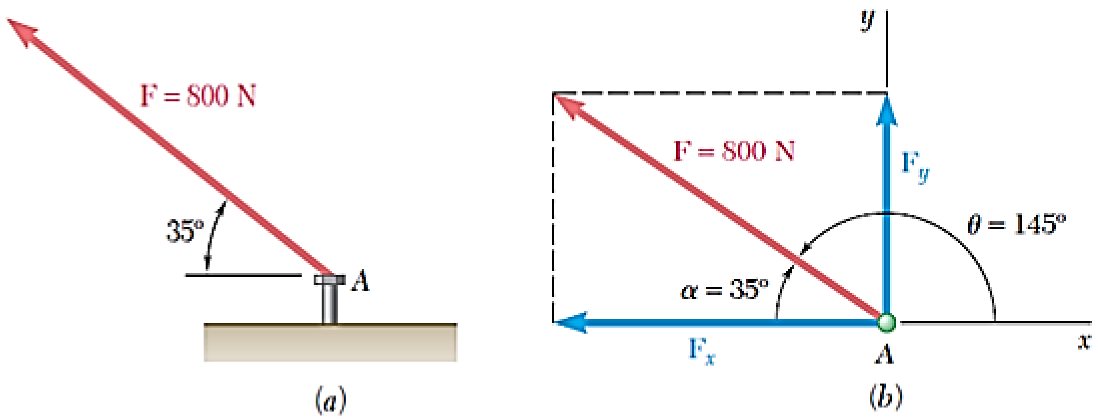
The components of **F**3 can be obtained by first computing the angle *α* of Fig. *c*.

$$\alpha = \tan^{-1}\left[\frac{0.2}{0.4}\right] = 26.6^\circ$$

$$F_{3x} = 800 \times \sin 26.6^\circ = 358N \longrightarrow$$

$$F_{3y} = 800 \times \cos 26.6^\circ = 716N \downarrow$$

Example: A force of 800 N is exerted on a bolt *A* as shown in Figure below. Determine the horizontal and vertical components of the force.



In order to obtain the correct sign for the scalar components *F x* and *F y* , the value θ = 180- 35 = 145 should be substituted.

$$F_y = F\,.\sin\theta \rightarrow F_y = 800.\,sin\,145 = 459\ N\ \uparrow$$

$$F_x = F\,.\cos\theta \rightarrow F_x = 800\,.\,cos\,145 = -655\ N \leftarrow$$

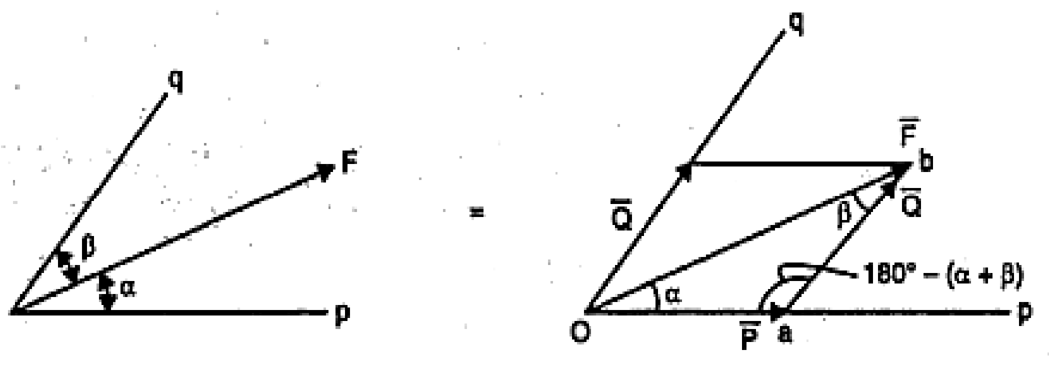
Or determine by inspection the signs of *F x* and *F y* and to use the trigonometric functions of the angle α= 35°.

$$F_y = +F\,.\sin\alpha \rightarrow F_y = +800.\,sin\,35^\circ = +459\ N\ \uparrow$$

$$F_x = -F\,.\cos\alpha \rightarrow F_x = -\,800\,.\,cos\,35^\circ = -655\ N \leftarrow$$

The Oblique forces (Resolving forces along two non-orthogonal components)(non- rectangular components)

So far forces have been resolved in two mutually perpendicular (or orthogonal) directions. But sometimes, it is required to resolve a given force into two given directions which are not perpendicular to each other. These directions are known as non-orthogonal directions.



(a) Given force

(b) Resolution of given force into components along p- and q- directions

A force F is given inclined as shown in figure(a) from the p- and q- directions. Let the required components be $\vec{P}$ and $\vec{Q}$, as shown in figure(b).The components of the force (F) can be determined graphically by drawing the parallelogram to any convenient scale. The magnitudes of The components can be determined algebraically from the law of sines & cosines. [the magnitude of the resultant force can be determined using the law of cosines, and its direction is determined from the law of sines. The magnitudes of two forces components are determined from the law of sines]. Then, by using the triangle law of forces, we get

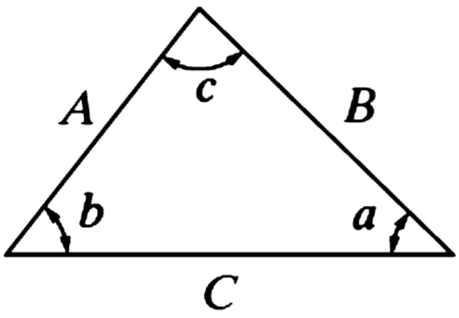
$$\frac{\vec{P}}{\sin\beta} = \frac{\vec{Q}}{\sin\alpha} = \frac{\vec{F}}{\sin(180^\circ - \alpha - \beta)}$$

$$\vec{P} = \vec{F}.\frac{\sin\beta}{sin(180^{\circ}-\alpha-\beta)}$$

$$\vec{Q} = \vec{F}.\frac{sin\,\alpha}{sin(180^{\circ}-\alpha-\beta)}$$

❖ Sine Law :

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$



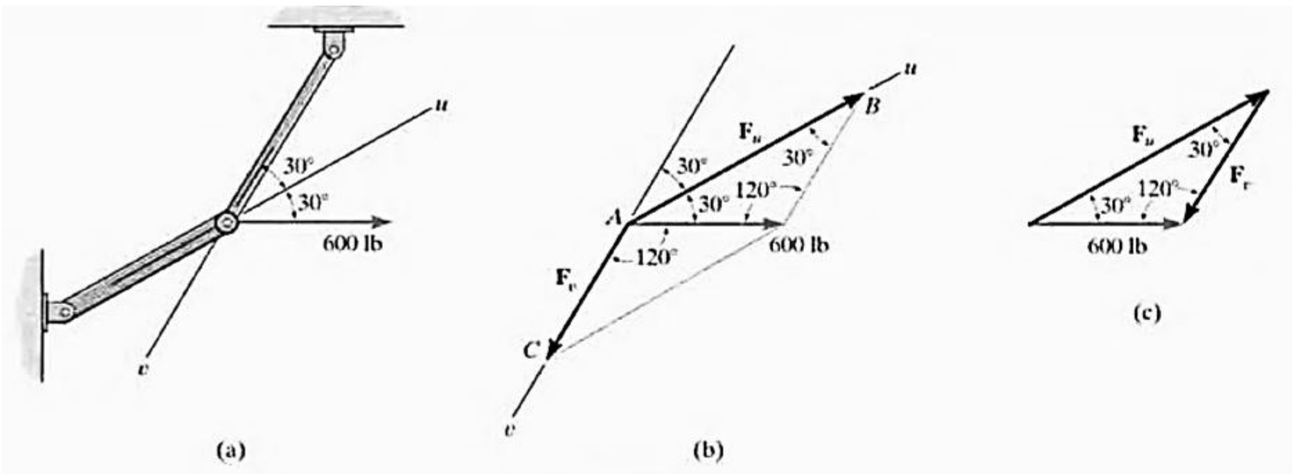
❖ Cosine Law:

$$A^2 = B^2 + C^2 - 2B.C\cos a$$

$$B^2 = A^2 + C^2 - 2A.C\ \cos b$$

$$C^2 = A^2 + B^2 - 2A.B\ \cos c$$

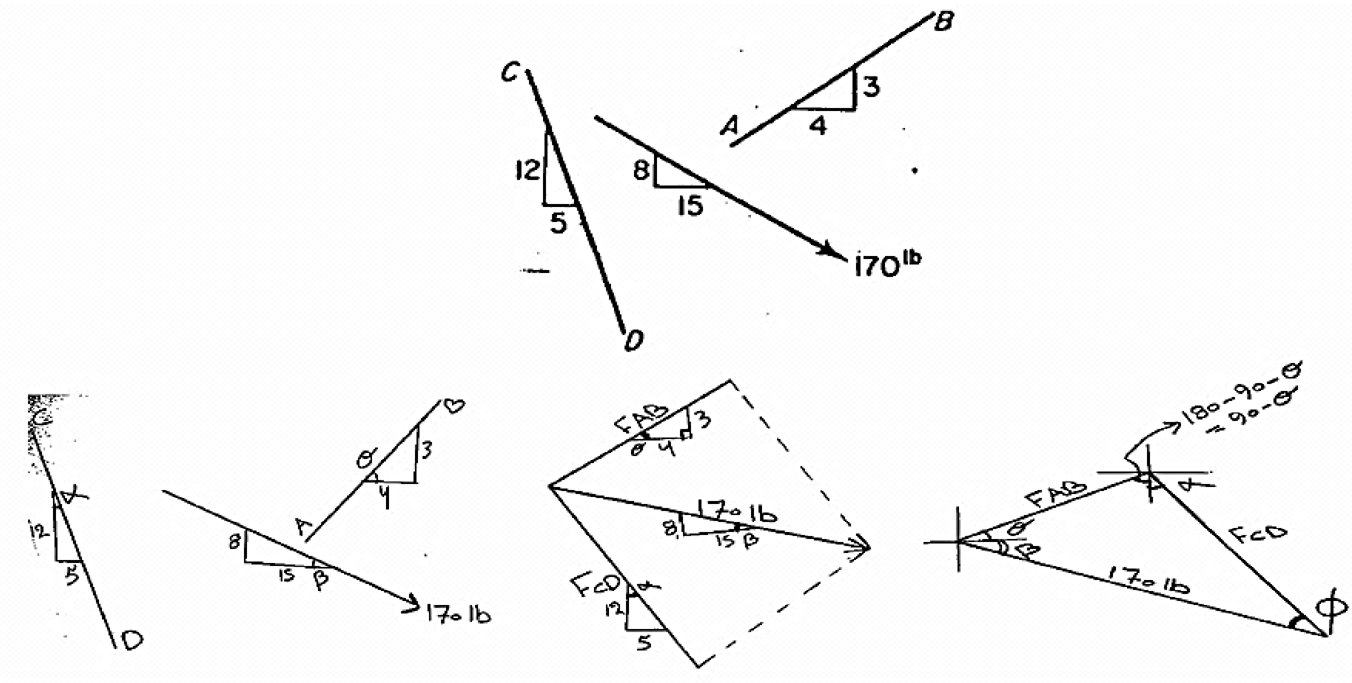
Example: Resolve the horizontal 600-lb force in figure below into components action along the u and v axes and determine the magnitudes of these components.



$$\frac{F_u}{\sin 120^{\circ}} = \frac{600lb}{\sin 30^{\circ}} \rightarrow F_u = 1039\text{ lb}$$

$$\frac{F_v}{\sin 30^{\circ}} = \frac{600lb}{\sin 30^{\circ}} \rightarrow F_v = 600lb$$

Example: Resolve the 170-lb force in figure below into two components, one along AB and the other parallel to CD.



$\theta = \tan^{-1}\frac{3}{4} = 36.87°$

$\beta = \tan^{-1}\frac{8}{15} = 28.07°$

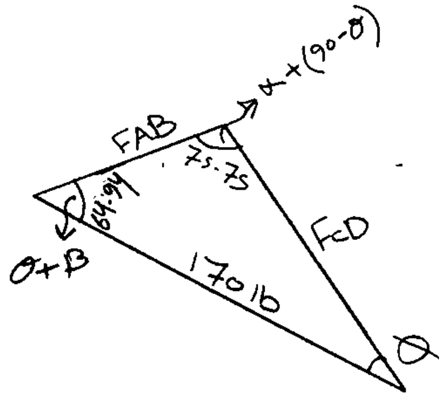
$\theta + \beta = 64.94°$

$\alpha = \tan^{-1}\frac{5}{12} = 22.62°$

$90 - \theta = 90° - 36.87° = 53.13°$

$\alpha + (90° - \theta) = 75.75°$

$\varphi = 180° - (64.94° + 75.75°) = 39.31°$



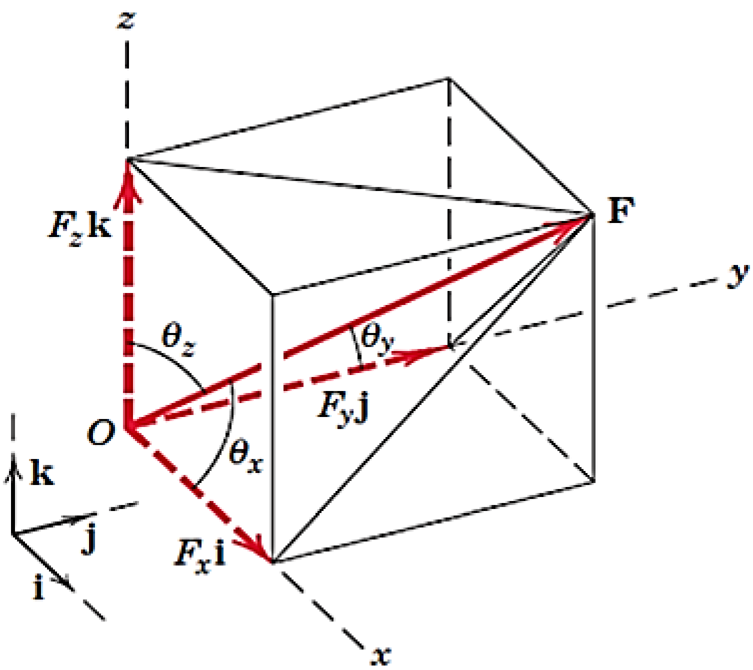
$$\frac{F_{AB}}{\sin 39.31°} = \frac{F_{CD}}{\sin 64.94°} = \frac{170^{\circ}}{\sin 75.75°}$$

$$\frac{F_{AB}}{sin\,39.31°} = \frac{170^{\circ}}{sin\,75.75°} \rightarrow\ F_{AB} = 111.11\ lb$$

$$\frac{F_{CD}}{sin\,64.94°} = \frac{170^{\circ}}{\sin 75.75°} \rightarrow\ F_{CD} = 158.89\ lb$$

B-Resolution of a force in space (3-D):

Many problems in mechanics require analysis in three dimensions, and for such problems it is often necessary to resolve a force into its three mutually perpendicular components. The force **F** acting at point *O* in Figure has the **rectangular components** F_x, F_y, F_z, where



$$F_x = F\cos\theta_x \qquad F = \sqrt{F_x{}^2 + F_y{}^2 + F_z{}^2}$$

$$F_y = F\cos\theta_y \qquad \mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k}$$

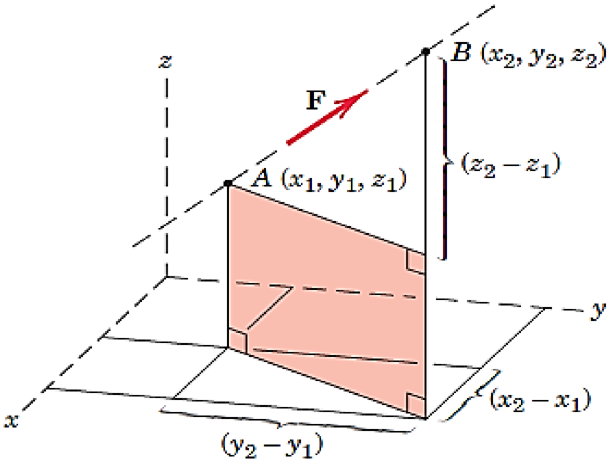
$$F_z = F\cos\theta_z \qquad \mathbf{F} = F(\mathbf{i}\cos\theta_x + \mathbf{j}\cos\theta_y + \mathbf{k}\cos\theta_z)$$

The unit vectors **i**, **j**, and **k** are in the *x*-, *y*-, and *z*-directions, respectively. The cosines of θ_x , θ_y , θ_z are known as the **direction cosines** of the force **F**.

- ❖ In solving three-dimensional problems, one must usually find the *x*, *y*, and *z* scalar components of a force. In most cases, the direction of a force is described (*a*) by two points on the line of action of the force or (*b*) by two angles which orient the line of action.

(a) Specification by two points on the line of action of the force.

If the coordinates of points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ are known, the force **F** may be written as:



$$F = F\frac{\overrightarrow{AB}}{AB} = F\frac{(x_2-x_1)i+(y_2-y_1)j+(z_2-z_1)k}{\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}}$$ Which mean that:

$$AB = r = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

$$F_x = F\frac{(x_2-x_1)}{r},\ F_y = F\frac{(y_2-y_1)}{r},\ \text{and}\ F_z = F\frac{(z_2-z_1)}{r}$$

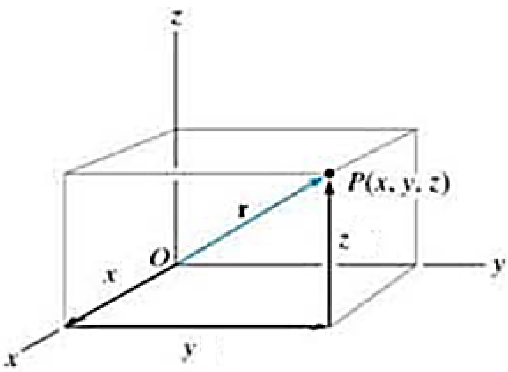
- ❖ If point A the origin O = (0,0,0) and B = p = (x,y,z) then the set of x,y,z components of the force may be written as:

$$r = \sqrt{x^2+y^2+z^2}$$

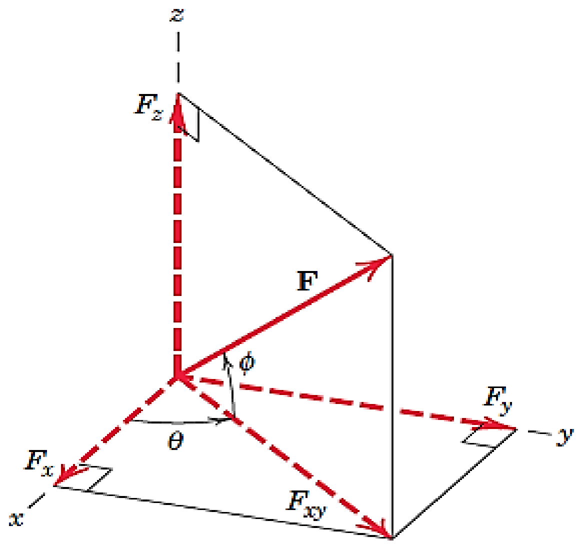
$$F_x = \frac{x}{r}\times F$$

$$F_y = \frac{y}{r}\times F$$

$$F_z = \frac{z}{r}\times F$$



(b) Specification by two angles which orient the line of action of the force. We assume that the angles θ and ϕ are known.



First resolve **F** into horizontal and vertical components.

$$F_{xy} = F \cos \phi$$
$$F_z = F \sin \phi$$

Then resolve the horizontal component F_{xy} into *x*- and *y*-components.

$$F_x = F_{xy} \cos \theta = F \cos \phi \cos \theta$$
$$F_y = F_{xy} \sin \theta = F \cos \phi \sin \theta$$

The quantities F_x, F_y, and F_z are the desired scalar components of **F**.

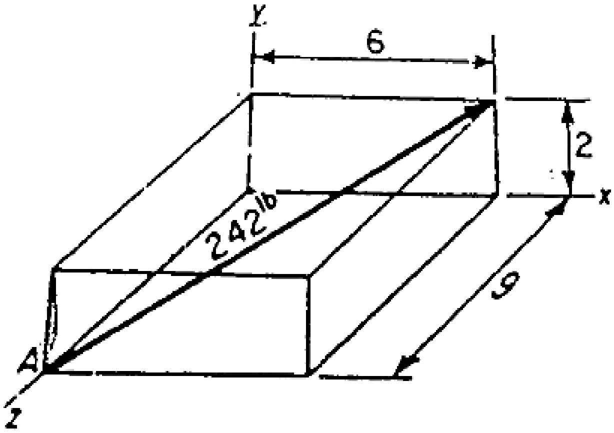
Example: determine a set of three rectangular components of the 242-lb force shown in figure below.

$$r = \sqrt{(6^2)+(2^2)+(9^2)} = \sqrt{121} = 11$$

$$F_x = \frac{6}{11} \times 242 = 132 \text{ lb}$$

$$F_y = \frac{2}{11} \times 242 = 44 \text{ lb}$$

$$F_z = \frac{9}{11} \times 242 = 198 \text{ lb}$$



Example: the 1000-lb force in figure below represents the force exerted on the rear wheel of an automobile in going around a curve. Determine a set of three rectangular components of the force. The (xy) plane is the horizontal plane.

First resolve F into vertical and horizontal components.

$$F_z = F \sin 75$$

$$F_z = 1000 \sin 75 = 966\ lb \uparrow$$

$$F_{xy} = F \cos 75 = 1000 \cos 75$$

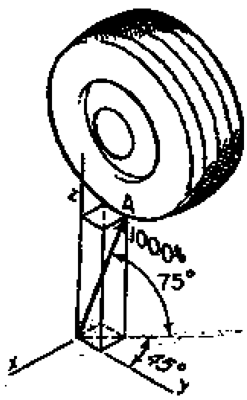
Then resolve the horizontal component F_{xy} into x- and y-components

$$F_x = F_{xy} \sin 45$$

$$F_x = 1000\ cos\, 75 \sin 45 = 183\ lb \nearrow$$

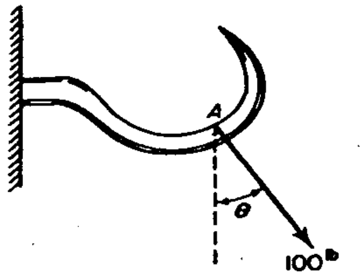
$$F_y = F_{xy} \cos 45$$

$$F_y = 1000\ cos\, 75 \cos 45 = 183\ lb \searrow$$

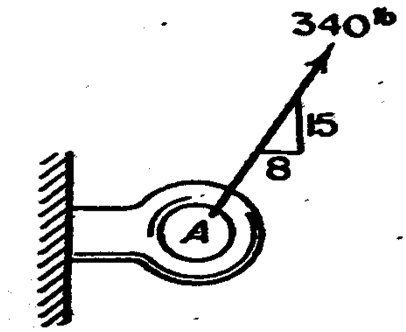


Home Work (2):

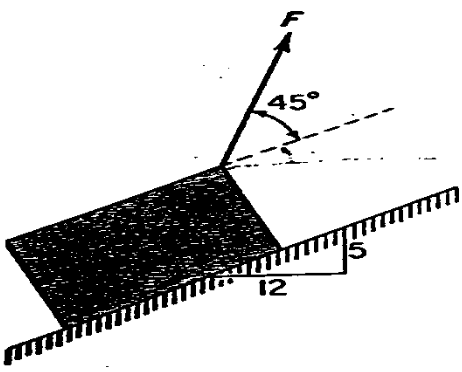
1- Resolve the 100-lb force of figure shown below into horizontal and vertical components for each of the following values of θ : (a) 22^o , (b) 78^o , (c) 132^o.



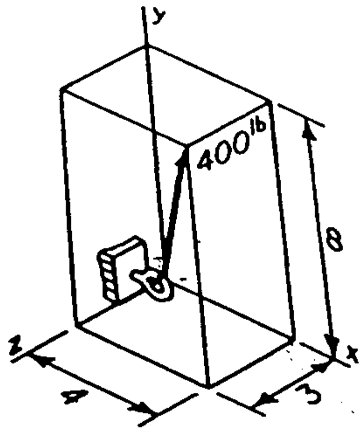
2- Determine a pair of horizontal and vertical components of the 340-lb force of figure shown below.



3- The force F which acts on the block of figure below has a horizontal rectangular component of 100-lb. Determine the rectangular component of F that is perpendicular to the inclined plane.



4- The tension in the rope attached to the eyebolt in figure below is 400-lb as shown. Determine a set of three rectangular components of the force.



5- Determine the components of the **F** force acting along the *u* and *v* axes. Given: θ1= 70^o , θ2= 45^o, θ3= 60^o, and F=250 N.