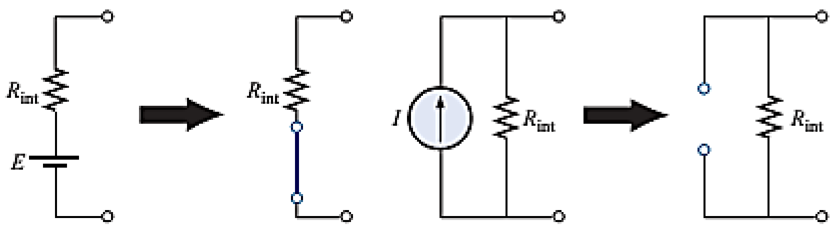
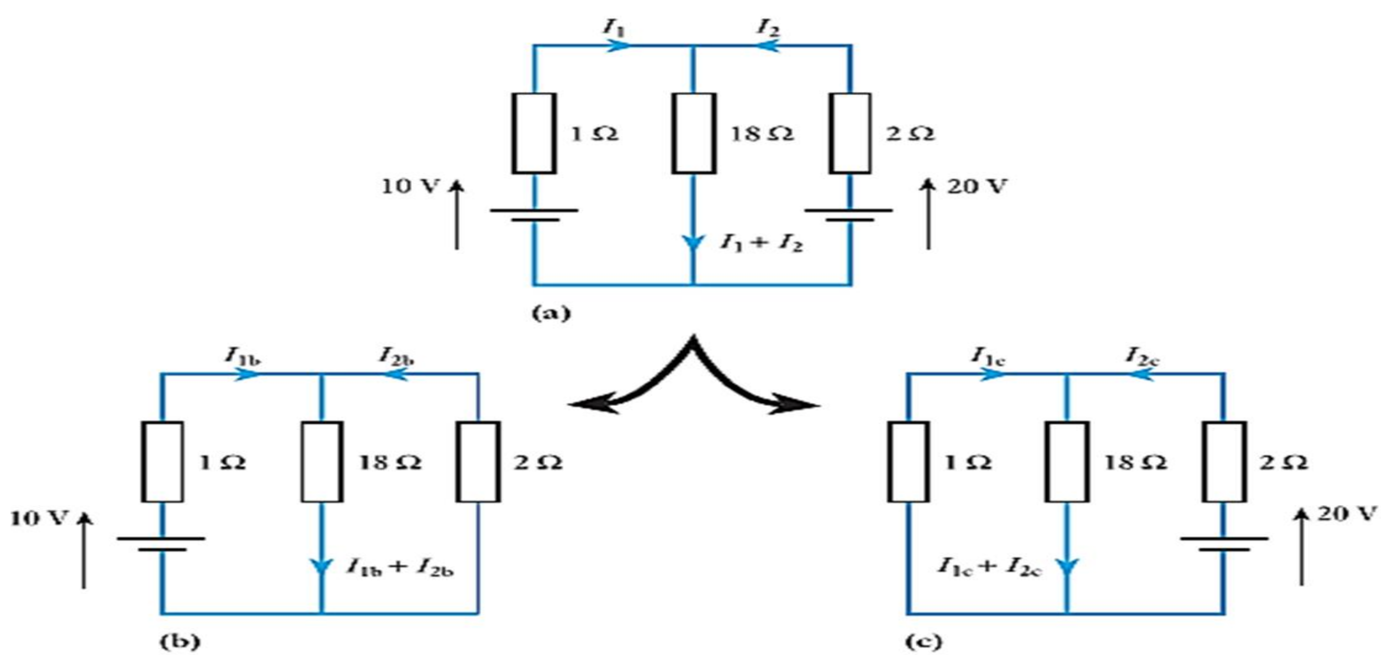
Superposition theorem:

The Superposition theorem states that in any network containing more than one source, the **current in**, or the **p.d. across**, any branch can be found by considering each source separately and adding their effects: omitted sources of e.m.f. are replaced by resistances equal to their internal resistances.



Example1:

By means of the Superposition theorem, calculate the currents in the network shown in Fig1



For the (b) arrangement, the total resistance RT is

$$1 + \frac{2 \times 18}{2 + 18} = 2.8\ \Omega$$

thus $$I_{1b} = \frac{10}{2.8} = 3.57\ \text{A}$$

and $$I_{2b} = -\frac{18}{2 + 18} \times 3.57 = -3.21\ \text{A}$$

also $$I_{1b} + I_{2b} = 3.57 - 3.21 = 0.36\ \text{A}$$

Note: the current I_{2b} is negative due to the direction in which it has been shown.

$$I_{2b} = \frac{I_T \times R_T}{R_2} = \frac{I_T}{R_2} \times \frac{R_1}{R_1 +} \frac{\times R_2}{R_2} = \frac{I_T \times R_1}{R_1 + R_2}$$

For the (c) arrangement, the total resistance RT is

$$2 + \frac{1 \times 18}{1 + 18} = 2.95\ \Omega$$

thus $$I_{2c} = \frac{20}{2.95} = 6.78\ \text{A}$$

and $$I_{1c} = -\frac{18}{1 + 18} \times 6.78 = -6.42\ \text{A}$$

$$I_{2c} + I_{1c} = 6.78 - 6.42 = 0.36\ \text{A}$$

Thus $$\mathbf{I_1 = I_{1b} + I_{1c} = 3.57 - 6.42 = -2.85\ A}$$

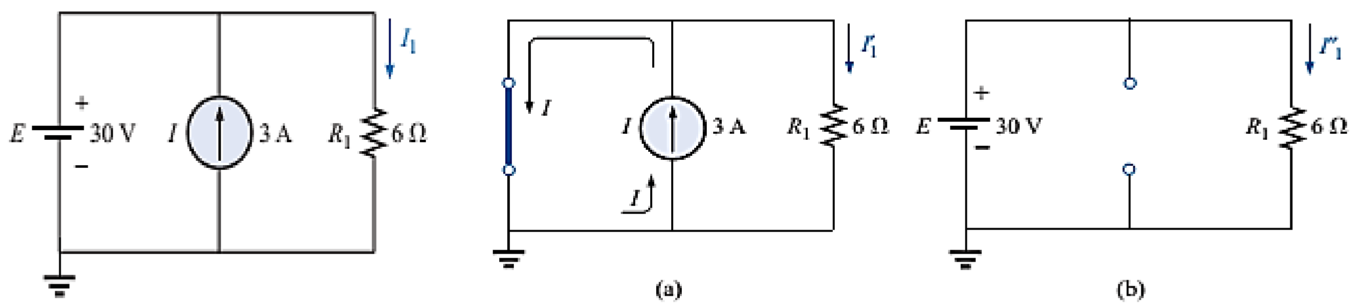
and $$\mathbf{I_2 = I_{2b} + I_{2c} = -3.21 + 6.78 = 3.57\ A}$$

also $$\mathbf{I_1 + I_2 = -2.85 + 3.57 = 0.72\ A}$$

EXAMPLE 2:

Determine I for the network of Fig2:

Solution: Setting E=0 V for the network of Fig2. Results in the network of Fig. 2(a), where a short-circuit equivalent has replaced the 30-V source.



As shown in Fig2 (a), the source current will choose the short circuit path, an

1=0A. If we applied the current divider rule,

$$I'_1 = \frac{R_{sc} I}{R_{sc} + R_1} = \frac{(0\ \Omega)I}{0\ \Omega + 6\ \Omega} = 0\ \text{A}$$

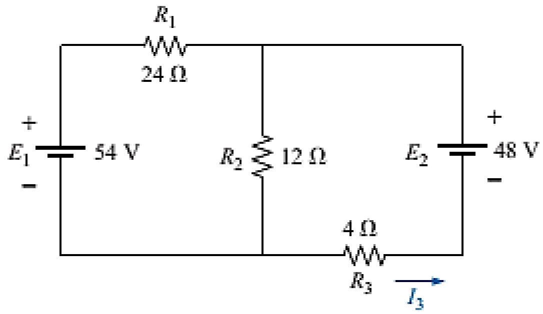
Setting I to zero amperes will result in the network of Fig2 (b), with the current source replaced by an open circuit. Applying Ohm

$$I''_1 = \frac{E}{R_1} = \frac{30\ \text{V}}{6\ \Omega} = 5\ \text{A}$$

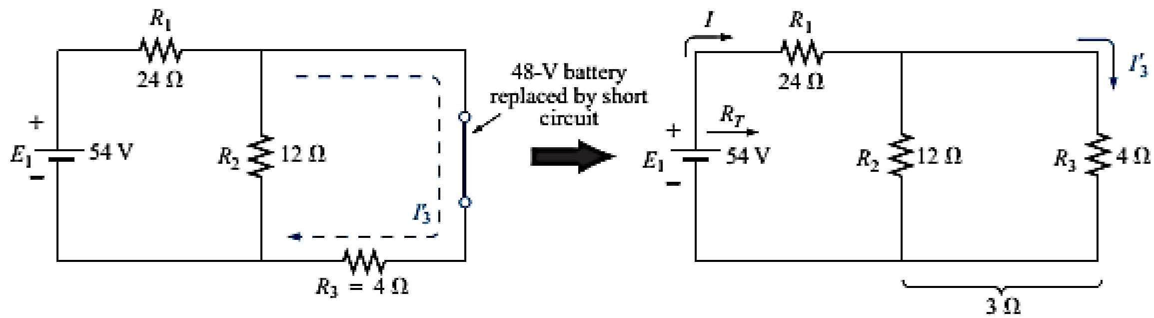
$$I_1 = I'_1 + I''_1 = 0\ \text{A} + 5\ \text{A} = \mathbf{5\ A}$$

EXAMPLE 3:

Using superposition, determine the current through the 4Ω resistor of Fig3.



Solution: Considering the effects of a 54-V source:



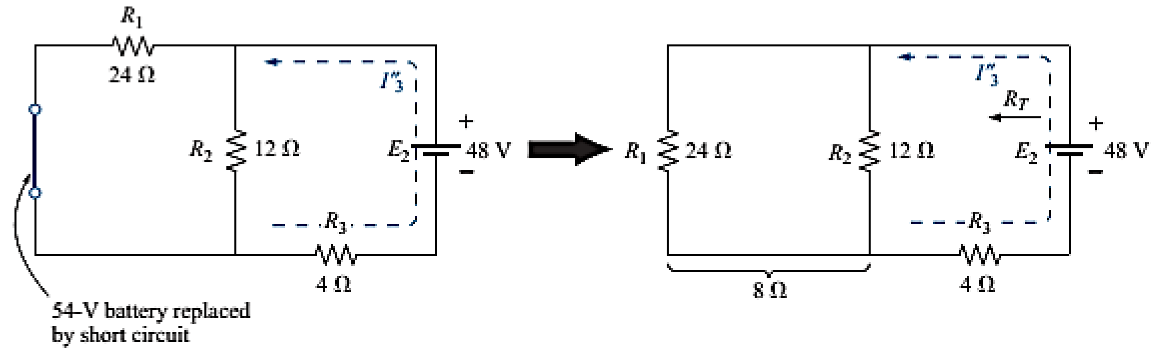
$$R_T = R_1 + R_2 \parallel R_3 = 24\ \Omega + 12\ \Omega \parallel 4\ \Omega = 24\ \Omega + 3\ \Omega = 27\ \Omega$$

$$I = \frac{E_1}{R_T} = \frac{54\ \text{V}}{27\ \Omega} = 2\ \text{A}$$

The effect of E_1 on the current I_3 Using the current divider rule

$$I'_3 = \frac{R_2 I}{R_2 + R_3} = \frac{(12\ \Omega)(2\ \text{A})}{12\ \Omega + 4\ \Omega} = \frac{24\ \text{A}}{16} = 1.5\ \text{A}$$

Considering the effects of the 48-V source

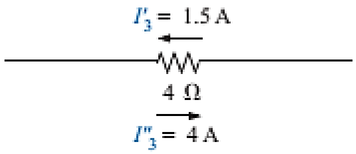


$$R_T = R_3 + R_1 \parallel R_2 = 4\ \Omega + 24\ \Omega \parallel 12\ \Omega = 4\ \Omega + 8\ \Omega = 12\ \Omega$$

$$I''_3 = \frac{E_2}{R_T} = \frac{48\ \text{V}}{12\ \Omega} = 4\ \text{A}$$

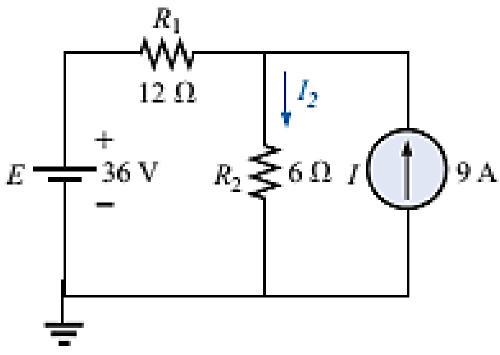
The total current through the 4Ω resistor is

$$I_3 = I''_3 - I'_3 = 4\ \text{A} - 1.5\ \text{A} = \mathbf{2.5\ A} \qquad (\text{direction of } I''_3)$$



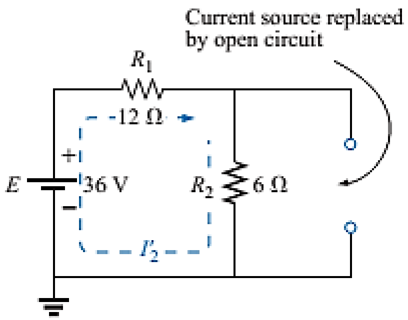
<u>EXAMPLE 4:</u>

Using superposition, find the current through the 6Ω resistor of the network of Fig4.



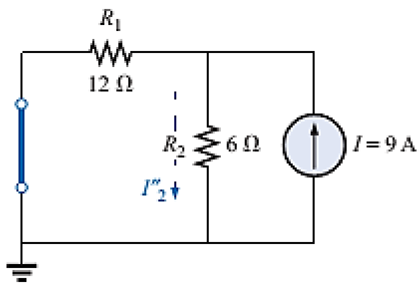
Solutions:

Considering the effect of the 36-V source:



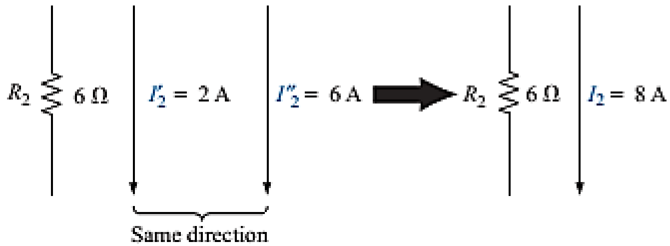
$$I'_2 = \frac{E}{R_T} = \frac{E}{R_1 + R_2} = \frac{36\text{ V}}{12\ \Omega + 6\ \Omega} = 2\text{ A}$$

Considering the effect of the 9-A source: Applying the current divider rule,



$$I''_2 = \frac{R_1 I}{R_1 + R_2} = \frac{(12\ \Omega)(9\text{ A})}{12\ \Omega + 6\ \Omega} = \frac{108\text{ A}}{18} = 6\text{ A}$$

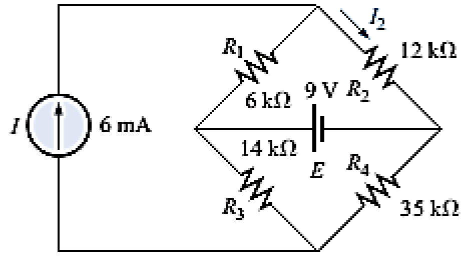
The total current through the 6Ω resistor is



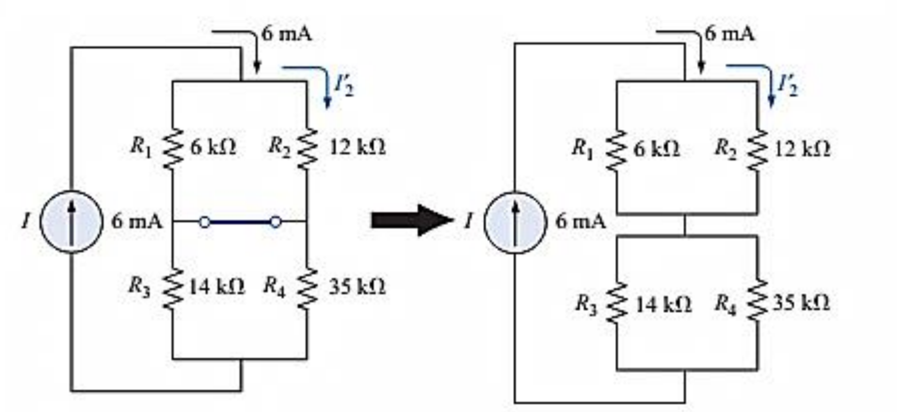
$$I_2 = I'_2 + I''_2 = 2\text{ A} + 6\text{ A} = \mathbf{8\ A}$$

EXAMPLE 5:

Using the principle of superposition, find the current I_2through the 12-kΩ resistor of Fig5



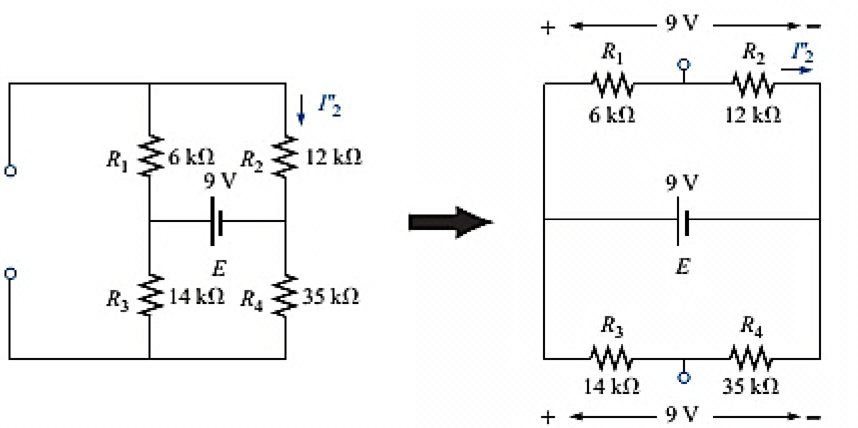
Solution: Considering the effect of the 6-mA current source:



Current divider rule:

$$I'_2 = \frac{R_1 I}{R_1 + R_2} = \frac{(6\ \text{k}\Omega)(6\ \text{mA})}{6\ \text{k}\Omega + 12\ \text{k}\Omega} = 2\ \text{mA}$$

Considering the effect of the 9-V voltage source:



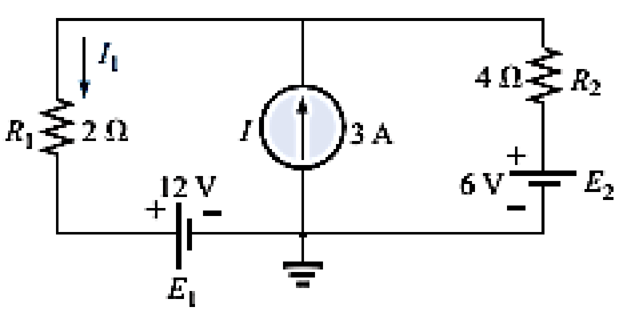
$$I''_2 = \frac{E}{R_1 + R_2} = \frac{9\text{ V}}{6\text{ k}\Omega + 12\text{ k}\Omega} = 0.5\text{ mA}$$

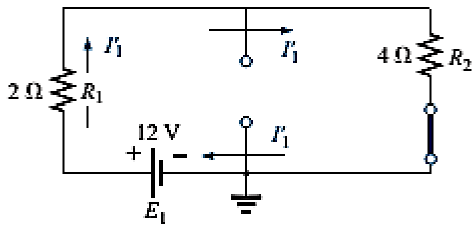
S i n c æ n d2 haI2e″the same direction through R2, the desired current is the sum of the two:

$$I_2 = I'_2 + I''_2$$
$$= 2\text{ mA} + 0.5\text{ mA}$$
$$= \mathbf{2.5\text{ mA}}$$

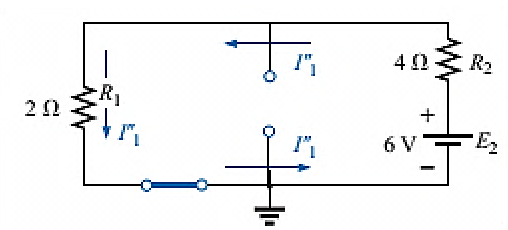
EXAMPLE 6:

Find the current through the 2Ω resistor of the network of Fig6. The presence of three sources will result in three different networks to be analyzed.

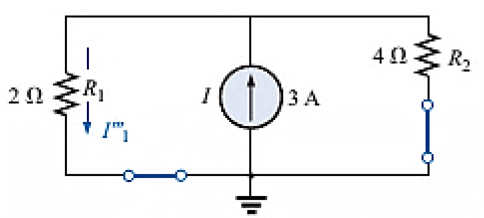




$$I'_1 = \frac{E_1}{R_1 + R_2} = \frac{12\,V}{2\,\Omega + 4\,\Omega} = \frac{12\,V}{6\,\Omega} = 2\,A$$



$$I''_1 = \frac{E_2}{R_1 + R_2} = \frac{6\,V}{2\,\Omega + 4\,\Omega} = \frac{6\,V}{6\,\Omega} = 1\,A$$

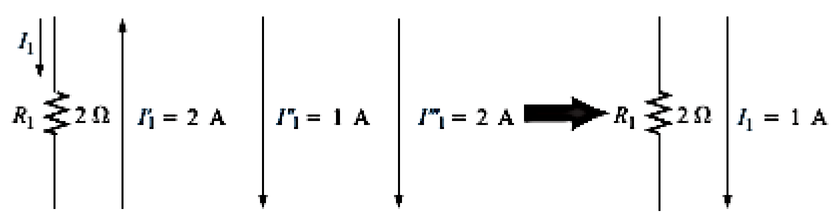


$$I'''_1 = \frac{R_2 I}{R_1 + R_2} = \frac{(4\,\Omega)(3\,A)}{2\,\Omega + 4\,\Omega} = \frac{12\,A}{6} = 2\,A$$

The total current through the 2Ω resistor appears

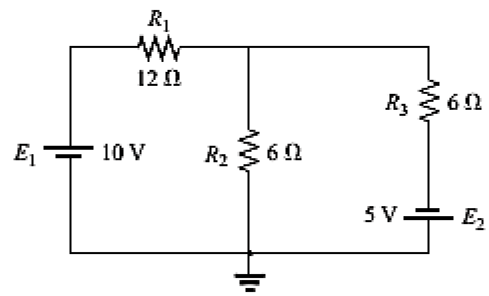
Same direction as I_1 in Fig. 9.19 — Opposite direction to I_1 in Fig. 9.19

$$I_1 = I''_1 + I'''_1 - I'_1$$
$$= 1A + 2A - 2A = 1A$$

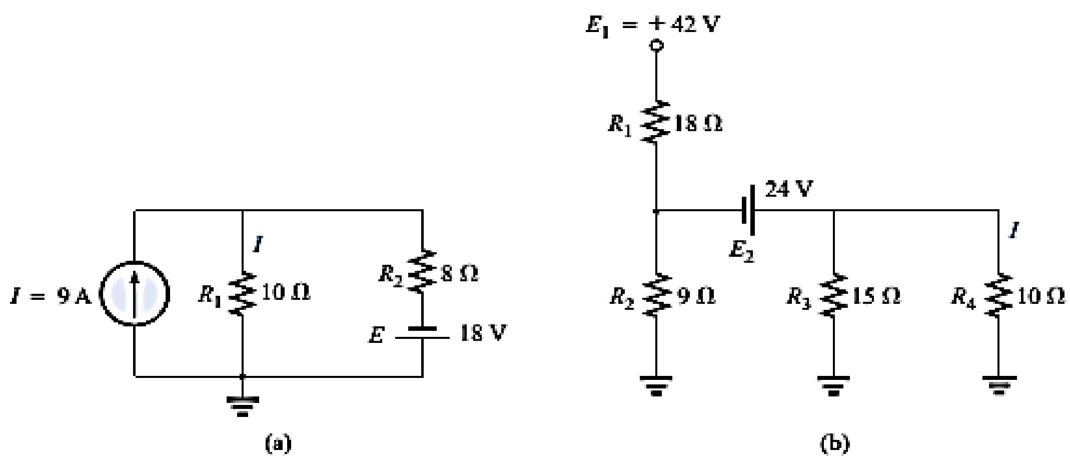


PROBLEMS

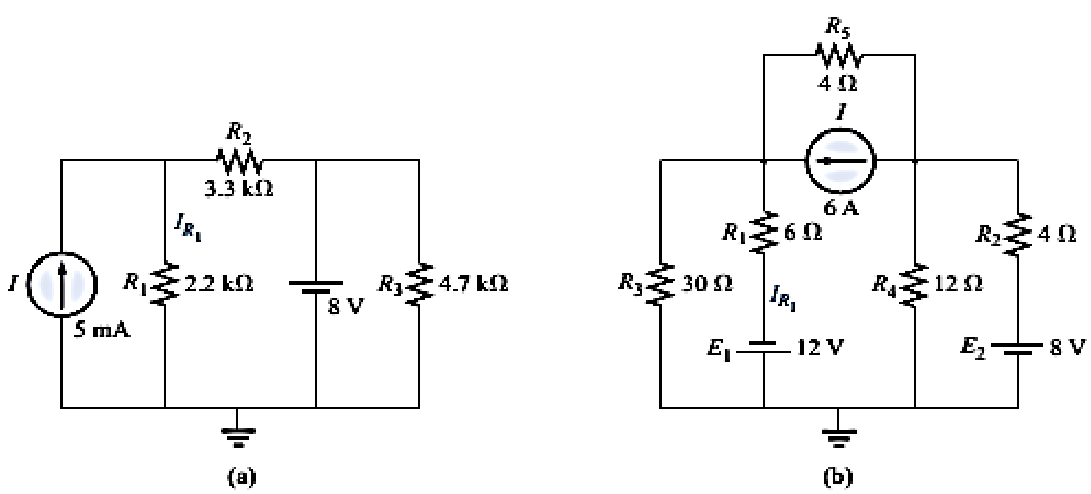
1. Using superposition, find the current through each resistor of the network of Fig. 1



2. Using superposition, find the current I



3. Using superposition, find the current through R_1 for each network of Fig3.



4. Using superposition, find the voltage $V_2$for the network of Fig4.