

Functions Continuity

①

A function $F(x)$ is said to be continuous at $x=a$ if :

- ① $F(a)$ exists (a lies on the domain)
- ② $\lim_{x \rightarrow a} F(x)$ exists ($F(x)$ has a limit as $x \rightarrow a$)
- ③ $\lim_{x \rightarrow a} F(x) = F(a)$ (the limit equals the function value)

Ex: is the function $f(x) = x+1$ continuous at $x=4$

Solution

$$① F(4) = 4+1 = 5$$

$$② \lim_{x \rightarrow 4} x+1 = 5$$

$$③ \lim_{x \rightarrow 4} x+1 = F(4) = 5$$

The function is continuous at $x=4$

EX: is the function continuous at $x=0$? ⁽²⁾

$$f(x) = \begin{cases} x^2 + 1 & x \geq 0 \\ x & x < 0 \end{cases}$$

solution

right side

$$f(x) = x^2 + 1, \quad f(0) = 0^2 + 1 = 1$$

$$\lim_{x \rightarrow 0} x^2 + 1 = 0^2 + 1 = 1$$

left side

$$f(x) = x, \quad f(0) = 0$$

$$\lim_{x \rightarrow 0} x = 0$$

limit from right \neq limit from left

The function is not continuous at $x=0$

