AL- ESRAA COLLEGE UNIVERSITY

Building & Construction Technology Engineering

Engineering Mechanics

First year

Resultant of Force Systems

By

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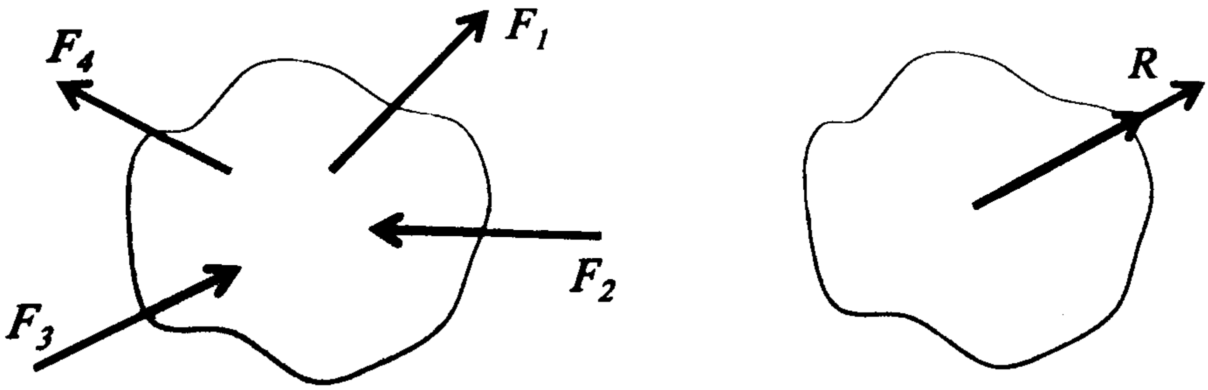
Resultant of Force Systems

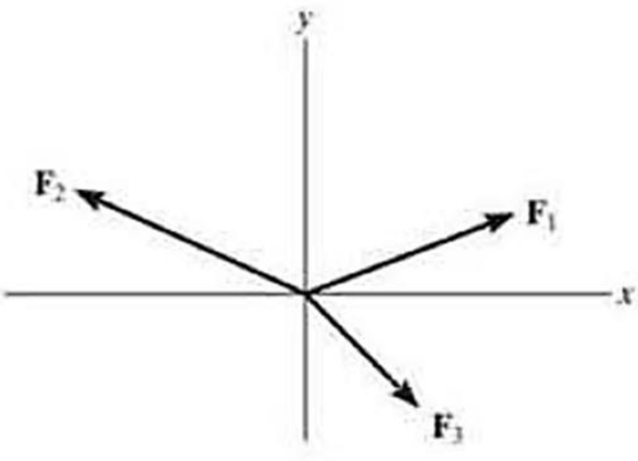
The resultant of a force system is the simplest force system which can replace the original system without changing its external effect on a rigid body.



- ❖ If R>0, the resultant will accelerate this body.
- ❖ If R=0, the body will remain in equilibrium or balanced state.

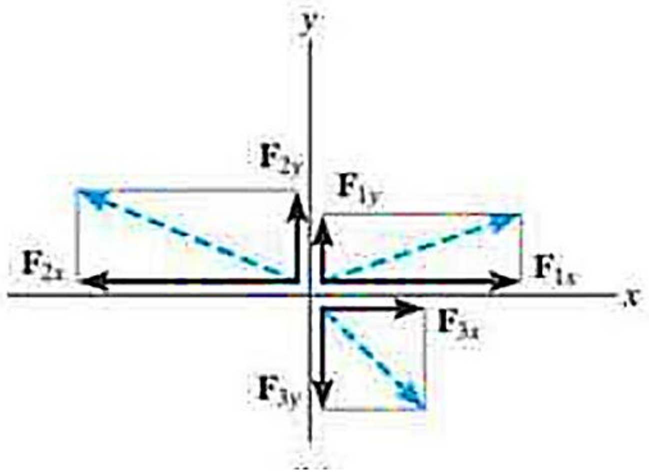
1- Resultant of Concurrent, Coplanar Force System.

In this system, line of action of all forces passes through a single point and forces lie in the same plane.



To find Resultant Force (R):

1- Resolving each force into its rectangular components,(Fy,Fx).

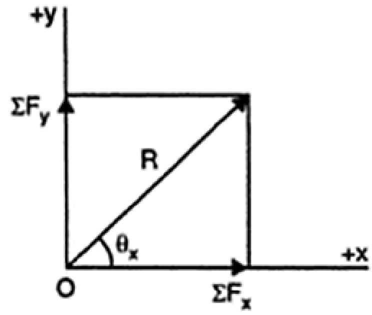


2- Find: $R_x = \sum F_x \rightarrow$, $R_y = \sum F_y \uparrow$

$R_x = F_{1x} - F_{2x} + F_{3x}$, $R_y = F_{1y} + F_{2y} - F_{3y}$

3- Find $R = \sqrt{R_x^2 + R_y^2}$

4- Find $\theta = \tan^{-1}\frac{R_y}{R_x}$ (direction of **R**)



Example: Determine the resultant of the concurrent, coplanar force system shown in figure.

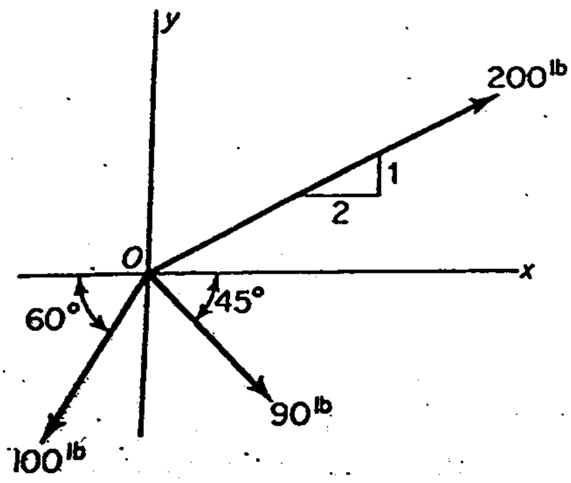
$R_x = \sum F_x \rightarrow$

$R_x = 200\left(\frac{2}{\sqrt{5}}\right) + 90\ \cos 45° - 100\ \cos 60°$

$R_x = 192.5\ lb \rightarrow$

$R_y = \sum F_y \uparrow$

$R_y = 200\left(\frac{1}{\sqrt{5}}\right) - 90\ \sin 45° - 100\ \sin 60°$

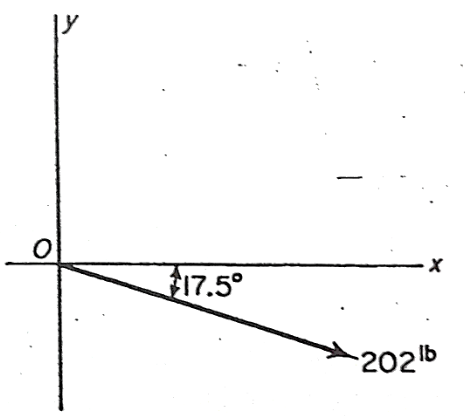


$R_y = -60.8\ \uparrow$

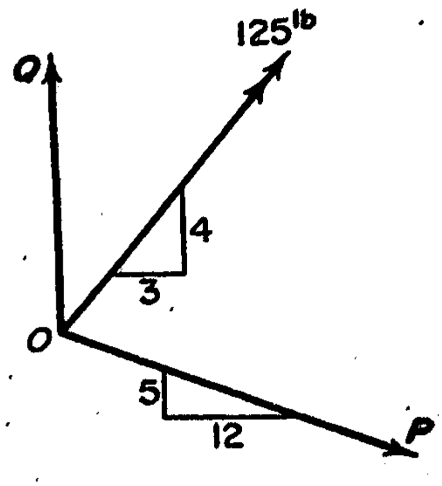
$R_y = 60.8\ lb\ \downarrow$

$R = \sqrt{R_x^2 + R_y^2} \rightarrow R = \sqrt{(192.5)^2 + (60.8)^2} = 202\ lb$

$\theta = \tan^{-1}\frac{R_y}{R_x} \rightarrow \theta = \tan^{-1}\frac{60.8}{192.5} = 17.5°$

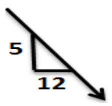


Example: The 125-lb force shown in figure below is the resultant of two forces **P** and **Q**. Determine the forces **P** and **Q**.



$R_x = \sum F_x\ \rightarrow$

$125\left(\frac{3}{5}\right) = p\left(\frac{12}{13}\right) \rightarrow p = 81.25\ lb$



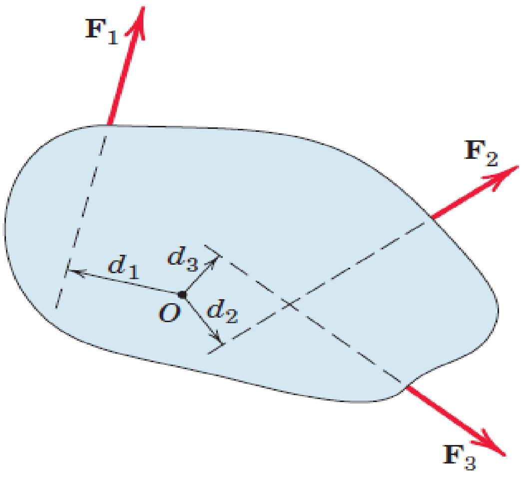
$R_y = \sum F_y\ \uparrow$

$125\left(\frac{4}{5}\right) = Q - p\left(\frac{5}{13}\right)$

$125\left(\frac{4}{5}\right) = Q - 81.25\left(\frac{5}{13}\right) \rightarrow Q = 131.25\ lb\ \uparrow$

2- Resultant of Nonconcurrent, Coplanar Force System.

All forces do not meet at a single point, but lie in a single plane. In this system forces divided in two:



a-Forces are not parallel

To determine the resultant of this case

1- Each force is resolved into rectangular components (Fx, Fy) in two directions arbitrarily called the x and y directions.

2- Find sum of the components in either the x and y direction.

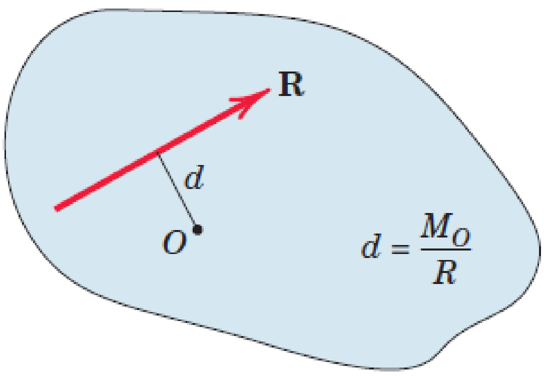
$$R_x = \sum F_x \rightarrow \;,\; R_y = \sum F_y \;\uparrow$$

3- Resultant magnitude can be determined from the following equation:

$$R = \sqrt{R_x^2 + R_y^2}$$

4- Find the angle between the x axis and the resultant from the relation

$$\theta_x = \tan^{-1}\frac{R_y}{R_x} \quad \text{(direction of } \mathbf{R}\text{)}$$



5- The location of the resultant is determined by the principle of moments (Varignon's theorem) , which states that the moment of the resultant about any point *O* equals the algebraic sum of the moments of the forces of the system with respect to the same point. Thus.

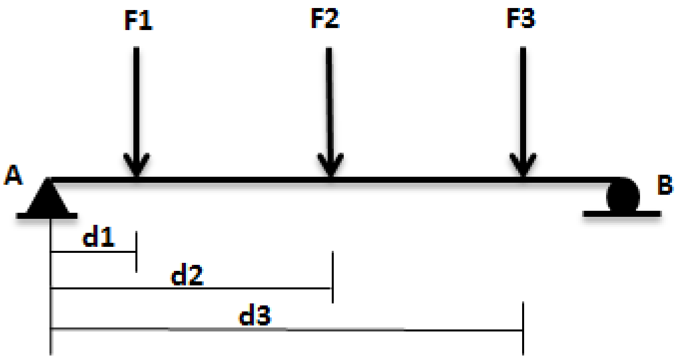
$$R.d = \sum M_o$$

$R.d = F_1 d_1 + F_2 d_2 + F_3 d_3$

$d = \frac{F_1 d_1 + F_2 d_2 + F_3 d_3}{R}$

b- Forces are parallel

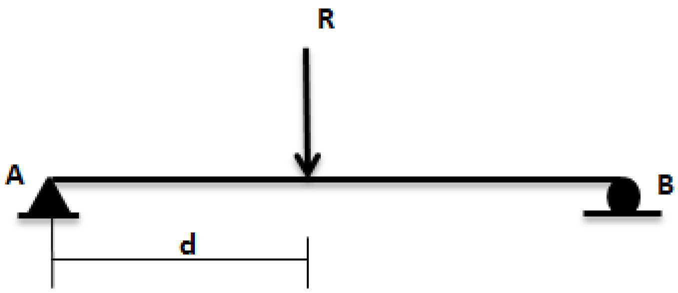
In this system, all forces are parallel to each other and lie in a single plane. The magnitude of the parallel resultant force R is simply the magnitude of the algebraic sum of the given forces ($= \sum F_y \downarrow$,or $R = \sum F_x \rightarrow$). For example the resultant of figure below can be found by:



$R = \sum F_y \downarrow$

$R = F_1 + F_2 + F_3$

To find the location of (R), [$R.d = \sum M_o$], the principle of moments (Varignon's theorem) is used. Where (d) is the perpendicular distance.



For example the resultant location of figure find by take moment about point A:

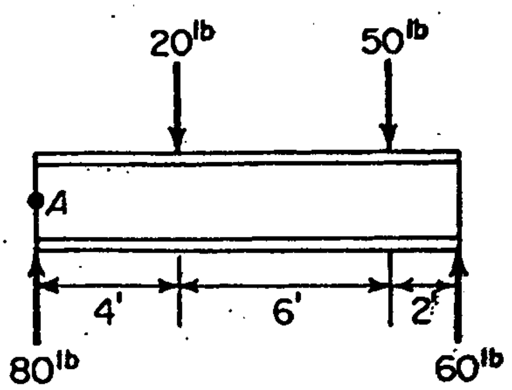
$\circlearrowright M_A = F_1 d_1 + F_2 d_2 + F_3 d_3$

$\circlearrowright M_A = R.d$

$R.d = F_1 d_1 + F_2 d_2 + F_3 d_3$

$d = \frac{F_1 d_1 + F_2 d_2 + F_3 d_3}{R}$

Example: Determine the resultant of the parallel, coplanar force system shown in figure, and locate it with respect to point A.



$R = \sum F_y \downarrow$

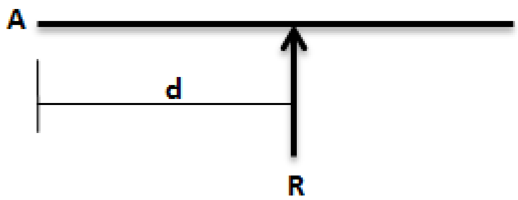
$R = 20 + 50 - 60 - 80 = -70 \text{ lb}$

$\uparrow R = 70 \text{ lb}$

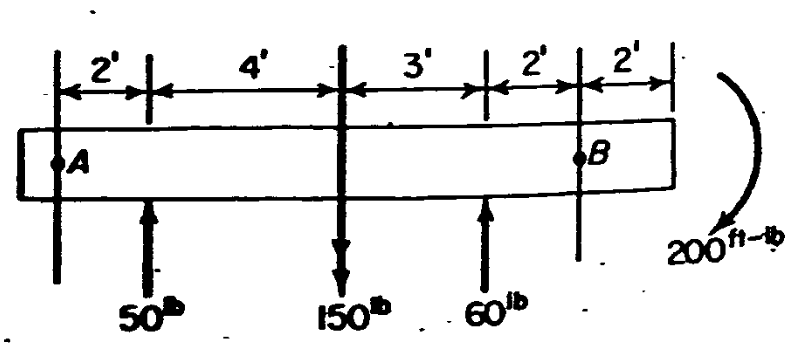
$R.d = \sum M_A$ ↻

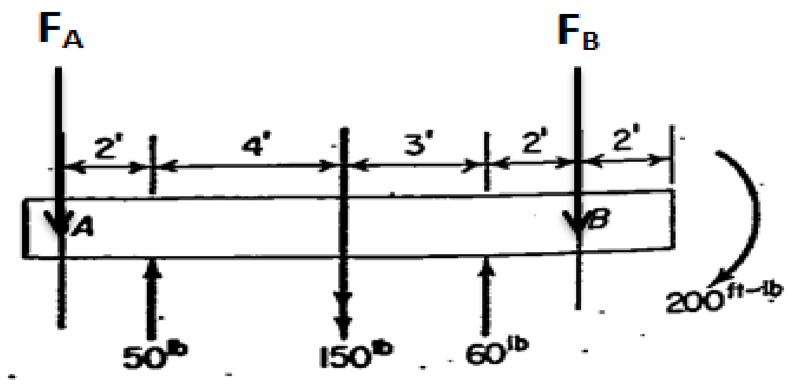
$(70)d = (60)(12) - (20)(4) - (50)(10)$

$d = \frac{140}{70} = 2 \text{ ft}$



Example: The 150-lb force of figure below is the resultant of the two forces and the couple and two other vertical forces, one acting through point A and the other through B. Determine these two unknown forces.





Let F_A , F_B unknown forces at point A,B

$$R.d = \sum M_A \circlearrowleft$$

$$(150)(6) = F_B(11) - (60)(9) - (50)(2) + 200$$

$$900 = (11)F_B - 440$$

$$F_B = 121.8\ lb\ \downarrow$$

$$R = \sum F_y \downarrow$$

$$150 = F_A - 50 - 60 + 121.8 \rightarrow \quad F_A = 138.2\ lb\ \downarrow$$

Example: Determine the resultant of the coplanar force system shown in figure, and locate it with respect to point O.

$$R_x = \sum F_x \rightarrow$$

$$R_x = (100)\left(\frac{3}{5}\right) - 130 = -70\ \text{lb}$$

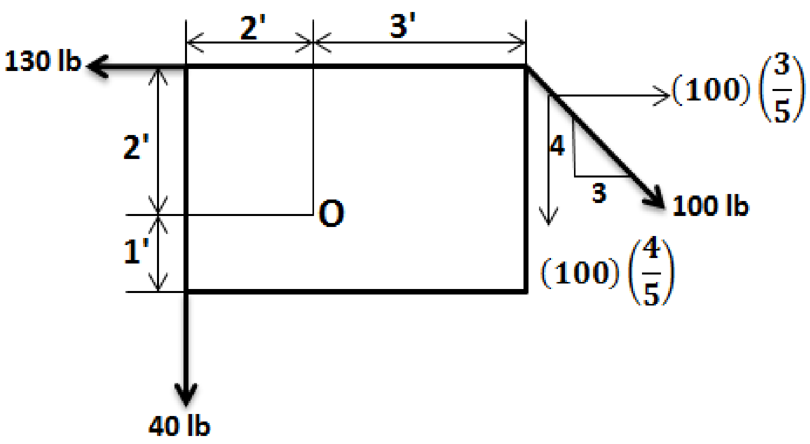
$$R_x = 70\ \text{lb} \leftarrow$$

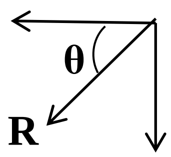
$$R_y = \sum F_y \downarrow$$

$$R_y = (100)\left(\frac{4}{5}\right) + 40 = 120\ lb \downarrow$$

$$R = \sqrt{R_x^2 + R_y^2}$$

$$R = \sqrt{(70)^2 + (120)^2} = 138.9\ lb$$

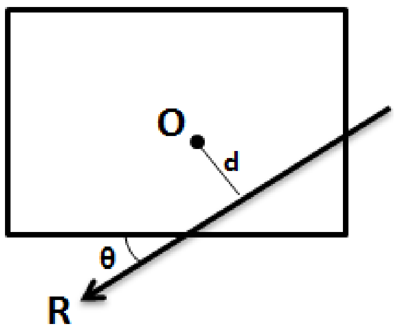




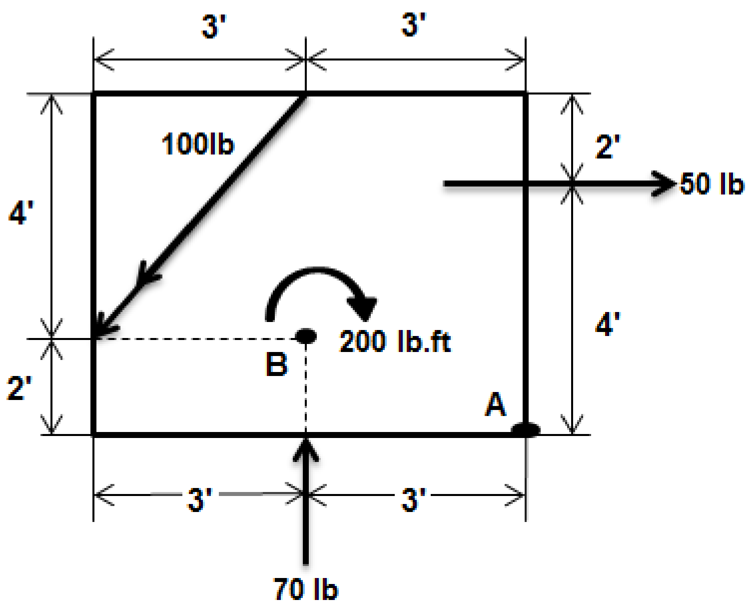
$\theta_x = \tan^{-1}\frac{R_y}{R_x} \rightarrow \quad \theta_x = \tan^{-1}\frac{120}{70} \rightarrow \quad \theta_x = 59.7°$

$R.d = \sum M_o$ ↶

$\therefore d = \frac{M_o}{R} = \frac{(100)\left(\frac{3}{5}\right)(2)+(100)\left(\frac{4}{5}\right)(3)-(130)(2)-(40)(2)}{(138.9)} = 0.14 \text{ ft}$



Example: The 100-lb force of figure below is the resultant of the couple and three forces, two of which are shown in the diagram. Determine the third force and locate it with respect to point A.



Let the third force is p

$R_x = (100)\left(\frac{3}{5}\right) = 60\ lb \downarrow$

$R_y = (100)\left(\frac{4}{5}\right) = 80\ lb \leftarrow$

$R_x = \sum F_x \leftarrow$

$60 = -50 + P_x$

$P_x = 110\ lb \leftarrow$

$R_y = \sum F_y \downarrow$

$80 = -70 + P_y$

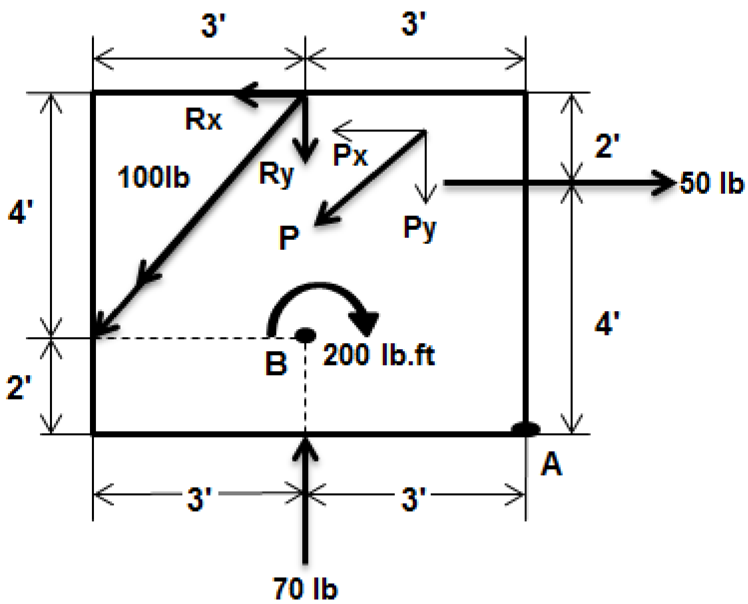
$P_y = 150\ lb \downarrow$

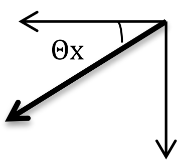
$P = \sqrt{P_x^2 + P_y^2}$

$P = \sqrt{110^2 + 150^2} = 186\ lb$

$\theta_x = \tan^{-1}\frac{P_y}{P_x}$

$\theta_x = \tan^{-1}\frac{150}{110} \rightarrow \theta_x = 53.7°$

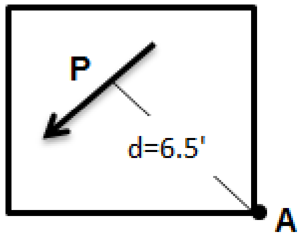




$R.d = \sum M_A$ ↻

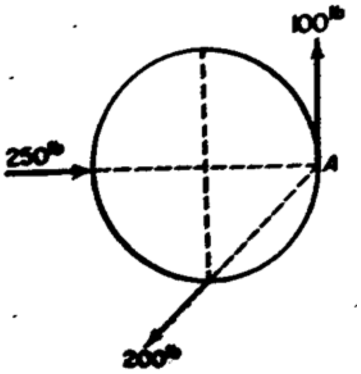
$(60)(6) + (80)(3) = (186)d - 200 - (50)(4) - (70)(3)$

$d = 6.5\ ft$

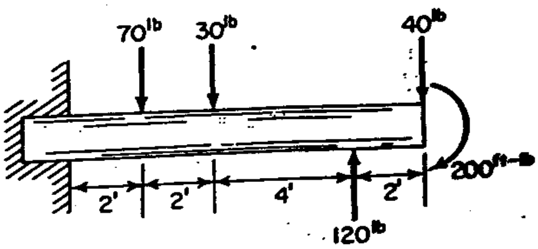


Home Work (5):

1- Determine the resultant of the concurrent, coplanar force system shown in figure.



2- Determine the resultant of the parallel, coplanar force system shown in figure.



3- Determine the resultant of the coplanar force system shown in figure, and locate it with respect to point A.