Flow of a Real Fluid

1. Introduction

The basic difference between an ideal fluid and a real fluid is that an ideal fluid is inviscid whereas a real fluid possesses viscosity.

Viscosity plays an important role in the motion of real fluid. Viscosity introduces resistance to motion by developing shear or frictional stress between the fluid layers and between the fluid layers and the boundary.

Existence of shearing resistance in real fluid causes the fluids to adhere to the solid boundary, and hence there is no relative motion between the fluid layers immediately in contact with the solid boundary and the solid boundary.

Viscosity causes the **flow** to occur in two different modes namely:

1. Laminar flow

2. Turbulent flow

Reynolds was first to distinguish between these two types of flow.

Reynolds established that at low velocities the fluid particles move in parallel layers and the flow is laminar. Laminar flow breaks down into turbulent flow at the upper critical velocity. If the velocity in a turbulent flow is reduced, the flow becomes laminar at the lower critical velocity.

* For pipe if $Re \leq 2000$ (the flow is laminar)

$Re > 4000$ (the flow is turbulent)

The **Reynolds number**, which is the ratio of the inertia (internal) force to the viscous (viscosity) force, shows the relative magnitudes of the inertia and viscous force.

$$\text{Reynolds number} = \frac{inertia\ force}{viscous\ force}$$

$$Re = \frac{\rho\, V\, D}{\mu} = \frac{V\, D}{\upsilon}$$

Where: $\upsilon = \frac{\mu}{\rho}$

Example 1

Find the largest velocity of water that can flow laminarly in a pipe of 110 mm diameter if the μ = 0.02 pa.s?

Solution

$$Re = \frac{\rho\, V\, D}{\mu}$$

$$2000 = \frac{1000 * V * 0.11}{0.02}$$

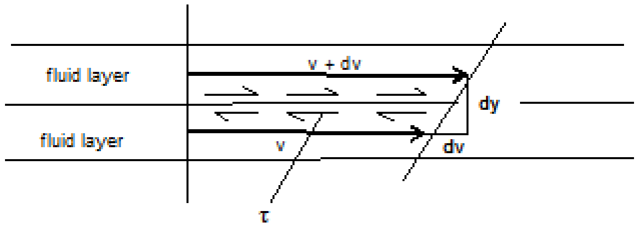
$$\therefore \mathrm{V} = 0.363\ \mathrm{m/s}$$

2. Shearing stresses for laminar and turbulent flow

In laminar flow, agitation of fluid particles is of molecular nature only, and these particles are constrained to motion in essentially parallel paths by the action of viscosity. The **shearing stress** between adjacent moving layers is determined in laminar flow by the viscosity and is completely defined by the differential equation.

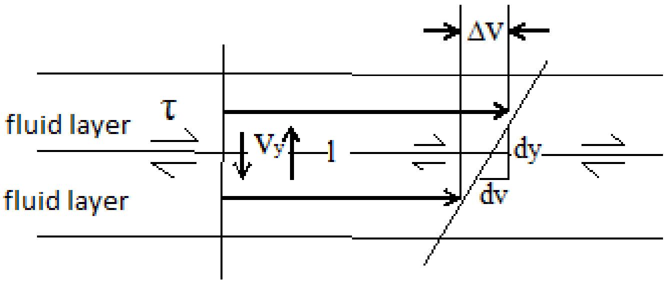
$$\tau = \mu \frac{dV}{dy}$$

The stress being the product of viscosity and velocity gradient (shown in figure below), if the laminar flow is disturbed by wall roughness or some other obstacle, the disturbances are rapidly are rapidly damped by viscous action and downstream the flow is smooth again. A laminar flow is stable against such disturbances, but a turbulent flow is not.



In turbulent flow, fluid particles do nor remain in layers, but move in a heterogeneous fashion through the flow, sliding past other particles and colliding with some in an entirely haphazard manner that result in a rapid and continuous mixing of the fluid as flow occurs. These particles are observed to travel in randomly moving fluid masses of varying sizes called **eddies**.

Shearing stress in turbulent flow:



$$\tau = \varepsilon \frac{dV}{dy}$$

Where:

ε = eddy viscosity depends on the flow characteristics.

$$\tau = (\mu + \varepsilon) \frac{dV}{dy}$$

* If the flow is laminar $\varepsilon = 0$

* If the flow is turbulent $\mu = 0$

$$\tau = \rho\, l^2 \left(\frac{dV}{dy}\right)^2$$

Where:

l = mixing length.

$$l = -k \frac{\left(\frac{dV}{dy}\right)^2}{\left(\frac{d^2V}{dy^2}\right)}$$

$$\tau = \rho\, k^2 \frac{\left(\frac{dV}{dy}\right)^4}{\left(\frac{d^2V}{dy^2}\right)^2}$$

Where, *k* = constant of the turbulence (Von –karman constant) (dimensionless)

Near the wall v_x or v_y approach to zero, one may suggests:

$l = k\,y$

So the equation $\tau = \rho\, l^2 \left(\frac{dV}{dy}\right)^2$ becomes:

$$\tau = \rho\, k^2\, y^2 \left(\frac{dV}{dy}\right)^2$$

Example 2

A turbulent flow of water occurs in a pipe of 2 m diameter. The velocity profile is measured experimentally and found to be closely approximated by the equation (v = 10 + 0.8 lny), in which v is in meters per second and y (the distance from the pipe wall) is in meters. The shearing stress in the fluid at a point 1/3 m from the wall is calculated analytically from measurements of pressure drop to be 103 pa. Calculate the eddy viscosity, mixing length, and turbulence constant at this point.

Solution

$$\frac{dV}{dy} = \frac{d}{dy}(10 + 0.8 \ln y) = \frac{0.8}{y} = \frac{0.8}{1/3} = 2.4$$

$$\frac{d^2V}{dy^2} = \frac{d}{dy}\left(\frac{dV}{dy}\right) = \frac{d}{dy}\left(\frac{0.8}{y}\right) = -\frac{0.8}{y^2} = -\frac{0.8}{\left(\frac{1}{3}\right)^2} = -7.2$$

$$\tau = \varepsilon \frac{dV}{dy}$$

103 = ε (2.4)

ε = 42.9 pa.s

$$\tau = \rho\, l^2 \left(\frac{dV}{dy}\right)^2$$

$103 = 1000\, l^2\, (2.4)^2$

$l = 0.134$ m

$$\tau = \rho\, k^2 \frac{\left(\frac{dV}{dy}\right)^4}{\left(\frac{d^2V}{dy^2}\right)^2}$$

$$103 = 1000\, k^2 \frac{(2.4)^4}{(-7.2)^2}$$

$k = 0.401$

or $\qquad \tau = \rho\, k^2\, y^2 \left(\frac{dV}{dy}\right)^2$

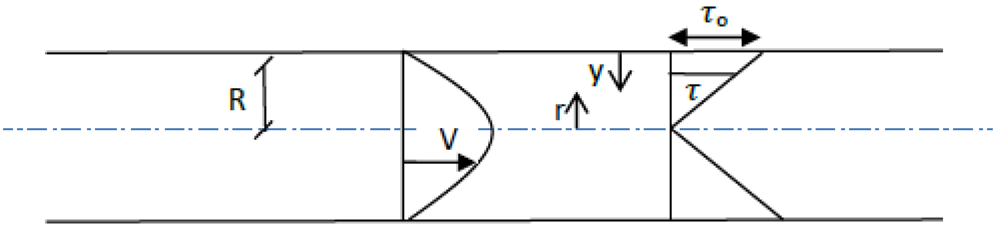
$$103 = 1000\, k^2\, (1/3)^2 (2.4)^2$$

$k = 0.401$

Example 3

Oil flows in a pipe has a diameter of 300 mm in laminar flow the velocity of oil at the center of the pipe is 4.5 m/s of the velocity distribution is parabolic profile (viscosity of oil = 1 pa.s). Find the shear stress at the wall and 50 mm from the wall pipe, $V = V_c\,(1- r^2/R^2)$ (V = velocity from center line to pipe, V_c = velocity at the center - max).

Solution



$dV/dr = -2\,(Vc/R^2)\,r$

$r = (R-y)$

The eq. of shear at laminar flow

$\tau = \mu\, dV/dy = -\mu\, dV/dr = (2\mu V_c/R^2)\, r$ [this eq. represent the straight line for shear stress]

Therefore, the shear stress at wall (r = R) equal to:

$\tau_o = (2\,\mu\, V_c / R^2)\, R \rightarrow \tau_o = (2*1*45) / 0.15 = 60$ pa

Also at the point 50 mm (y = 50 mm) from the wall

<u>or</u> at 100 mm (r = 150-50 = 100 mm) from the center

$\tau_o/R = \tau/r$

$\tau = \tau_o\,(r/R)$

$\tau = \frac{2}{3}\,\tau_o = 40$ pa

<u>or</u> applied the above equation

$\tau_{50} = 2\,\mu\, V_c/R^2 * r$

r = 100 mm = 0.1 m

Example 4

An oil (r.d = 0.9) flows in pipe (600 mm) in turbulent flow and the velocity distribution can be as formatted by the equation: $V = 3.56\, y^{1/7}$.

If the shear stress in oil at 150 mm from the wall is equal to 12.44 pa, find the eddied viscosity and the mixing length and coefficient of turbulence at this point.

Solution

At 150 mm from the wall

At $y = 0.15 \rightarrow dV/dy = 1/7\,(3.56)\,y^{-6/7} \rightarrow dV/dy = 2.585$

& $d^2V/dy^2 = 1/7\,(3.56)\,(-6/7)\,y^{-13/7} = -14.77$

From the equation of shear stress for turbulent flow

$T = \varepsilon\, dV/dy = 12.44 = \varepsilon\,(2.585)$

$\varepsilon = 12.44/2.585 = 4.812$ pa.s

And from the equation related the shear stress with turbulent mixing length for turbulent flow

$$\tau = \rho\, l^2 \left(\frac{dV}{dy}\right)^2$$

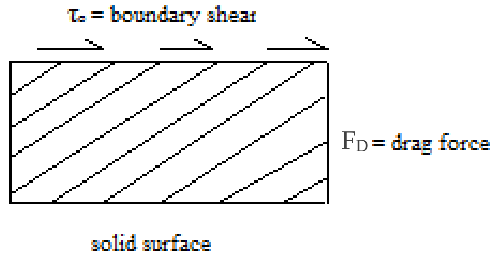
$12.44 = 0.9*1000\, l^2 *(2.585)^2 \quad \rightarrow \; l = 0.04$ m or 45 mm

And from eq. van - karman

$$\tau = \rho\, k^2 \frac{\left(\frac{dV}{dy}\right)^4}{\left(\frac{d^2V}{dy^2}\right)^2}$$

$k = 0.26$

3. Flow of Fluid over Solid Surfaces (Flow resistance)



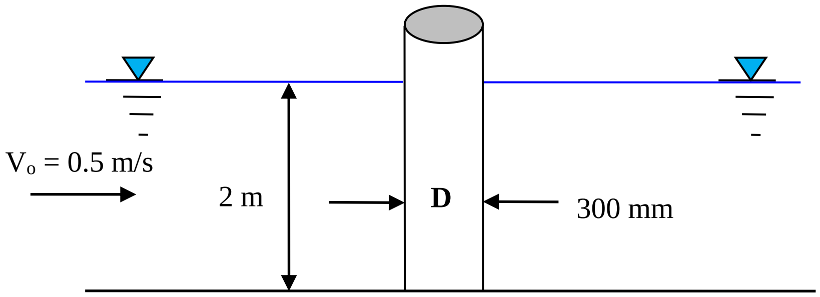
The force exerted by the fluid on the surface F_D

F_D= f (L , S_f , e , V_o , ρ , μ)

$$F_D = C_D \frac{A\,\rho\,V_o^2}{2}$$

Example 5

Find the drag force on the column shown in figure. Where C_D = 1.2, υ = 1.139 * 10^{-6}.



Solution

$$Re = \frac{V\,D}{\upsilon} = \frac{0.5 * 0.3}{1.139 * 10^{-6}} = 1.317 * 10^{6}$$

$$F_D = C_D \frac{A\,\rho\,V_o^2}{2} = 1.2\,\frac{(2 * 0.3) * 1000 * 0.5^2}{2} = 90\ \text{N}$$

4. Energy loss in viscous flow

$$\frac{p_1}{\gamma} + \frac{V_1^{\,2}}{2\,g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^{\,2}}{2\,g} + z_2 + h_f$$

$$h_f = \frac{\tau_o\, L}{\gamma\, R_h}$$

$R_h = A/P$, $h_f/L = S_e$

$\tau_o = \gamma\, R_h\, S_e$

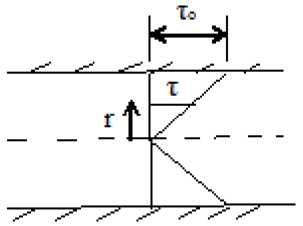
p = wetted perimeter, R_h= hydraulic radius,

S_e = slope of energy line.

Substitute τ for τ_o and $r/2$ for R_h in the above equation:

$$\tau = \frac{\gamma\, h_f}{2\, L}\, r$$

Which is the equation of the straight line.



Example 6

Liquid (r.d = 0.9) flow in pipe of 200 mm diameter which is placed horizontally. The velocity distribution is given by ($V = 100 - r^2$) where r is the distance from the center (cm) and V is the velocity (cm/s) at distance r from the center. If the viscosity μ = 0.12 pa.s, find: (a) The discharge, average velocity and the velocity a distance 50 mm from the center, (b) The shearing stress at the wall and at a distance 30 mm from the wall and draw the distribution of the shearing stress. (c) Energy loss in 1000 m of the flow , (d) Power needed to move the liquid 1000m , (e) If you double the discharge, what happen to the answer in (a),(b),(c) and (d).

Solution

$$(a)\ \ Q = \int_0^R V\,dA = \int_0^{10} (100 - r^2)\, 2\,\pi\, r\, dr = 2\,\pi \int_0^{10} (100\, r - r^3)\, dr$$

$$= 2\,\pi \left[50\, r^2 - \frac{r^4}{4} \right]_0^{10} = 15.708\ L/s$$

$$V = \frac{Q}{A} = \frac{0.015708}{\pi\,(0.1)^2} = 0.5\ m/s$$

$V_{r\,=\,50\,mm}$ = 100 – $(5)^2$ = 75 cm/s = 0.75 m/s

$$(b)\ \ Re = \frac{\rho\, V\, D}{\mu} = \frac{(0.9 * 1000)\,(0.5)\,(0.2)}{0.12} = 750 \quad (laminar\ flow)$$

$$\tau = \mu\, \frac{dV}{dy} \quad , \quad dy = -\,dr \quad , \qquad \therefore\ \tau = -\,\mu\, \frac{dV}{dr}$$

$$\frac{dV}{dr} = -2\, r \quad , \quad so \quad \tau = 2\,\mu\, r$$

τ_o = 2 μ r = (2) (0.12) (0.1) = 0.024 pa

$\tau_{y\,=\,30\,mm}$ = 2 μ r = (2) (0.12) (0.07) = 0.0168 pa

$$(c)\ \ h_f = \frac{\tau_o\, L}{\gamma\, R_h} = \frac{(0.024)\,(1000)}{(0.9 * 9.8 * 1000)\,(0.05)} = 0.05442\ m$$

(d) Pr = Q ɤ hf = (0.015708) (0.9 * 9800) (0.0544) = 7.59 w

or

Pr = F V = ($_o$ p L) (V) = (0.024) (π*0.2 m) (1000) (0.5) = 7.59 w

(e) Doubling the discharge will double the average velocity so Reynolds number will double but the flow will remain laminar. And the velocity distribution will remain parabola so the velocity will double at all locations, shearing stress will double as a result head loss will double but power will be four times larger.