

Differentiation

If a function (f) is continuous on (a, b) then

$$\lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \text{ exists at point } (a, b)$$

We call this limit the derivative or the slope of the function and denoted by

$$f'(x) \text{ or } \frac{df(x)}{dx} \text{ or } \frac{dy}{dx} \text{ or } y'$$

The derivative of a function at a point $x=a$, is the slope of the tangent line to the curve of the function (f) at this point.

Ex: Use the definition to find the slope of $y = 2x + 2$ at $x = 4$

Solution: $\lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{2(4+\Delta x) + 2 - (2 \cdot 4 + 2)}{\Delta x}$

$$\lim_{\Delta x \rightarrow 0} \frac{2(4+\Delta x) + 2 - (8 + 2)}{\Delta x} = \frac{8 + 2\Delta x + 2 - 8 - 2}{\Delta x} = 2$$