AL- ESRAA COLLEGE UNIVERSITY

Building & Construction Technology Engineering

Engineering Mechanics

First year

Equilibrium

By

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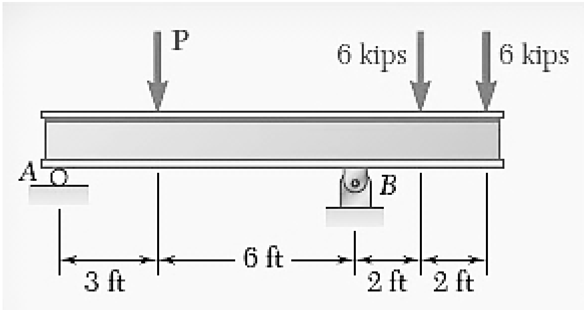
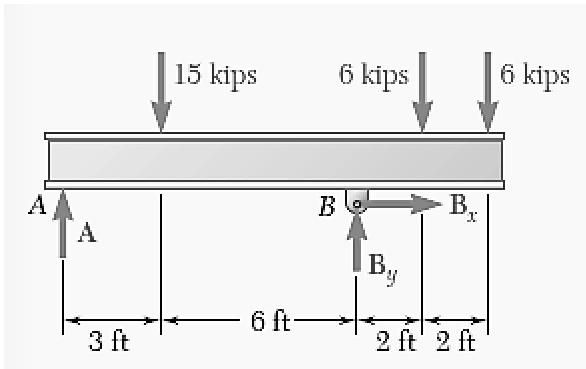
<u>Example:</u> Three loads are applied to a beam as shown. The beam is supported by a roller at *A* and by a pin at *B*. Neglecting the weight of the beam, determine the reactions at *A* and *B* when p= 15 kips.



1-Free-Body Diagram: A free-body diagram of the beam is drawn.



2- **Equilibrium Equations:** We write the following three equilibrium equations and solve for the reactions indicated:

$\sum F_x = 0 \ \rightarrow, \quad B_x = 0$

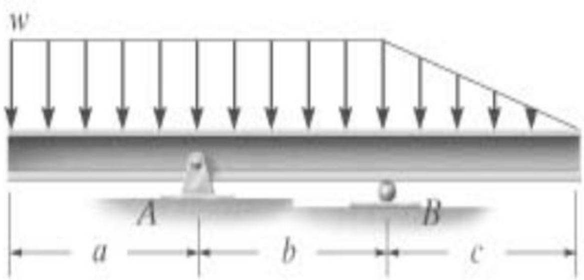
$\sum M_A = 0$ ↶+

$\sum M_A = -(15)(3) - (6)(11) - (6)(13) + B_y(9) = 0 \rightarrow \mathrm{B_y} = 21 \text{ Kips} \uparrow$

$\sum F_y = 0 \ \uparrow$

$\sum F_y = A + 21 - 15 - 6 - 6 = 0 \rightarrow A = 6\, Kips \ \uparrow$

Example: Determine the reactions at the supports. Given: (w=250 lb/ft, a=6 ft, b=6ft, c=6ft)



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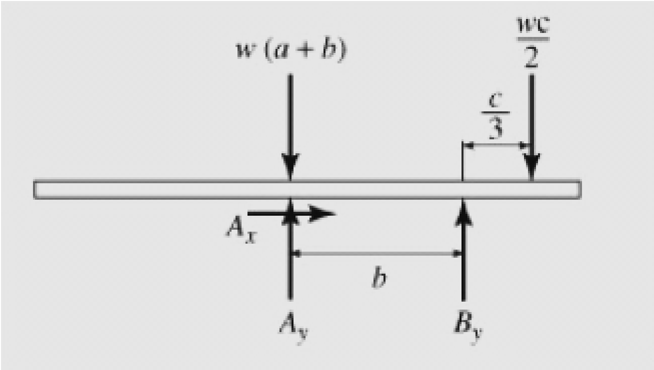


2- **Equilibrium Equations:** We write the following three equilibrium equations and solve for the reactions indicated:

$\sum F_x = 0 \rightarrow, \quad A_x = 0$

$\sum M_A = 0$ ↺+

$B_y(b) - \frac{w_c}{2}\left(\frac{c}{3} + b\right) = 0$

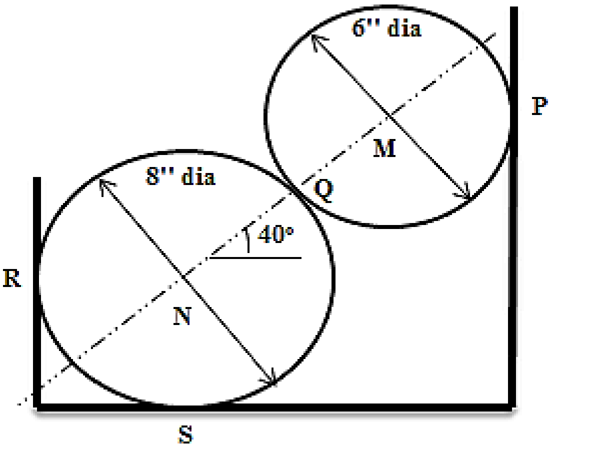
$B_y(6) - \frac{250*6}{2}(2+6) = 0 \rightarrow B_y = 1000lb \uparrow$

$\sum F_y = 0 \uparrow \quad \rightarrow A_y + B_y - w(a+b) - \frac{wc}{2} = 0$

$A_y + 1000 - 250(6+6) - \frac{250*6}{2} = 0$

$A_y = 2750\ lb\ \uparrow$

Example: the 300lb shaft M and the 500lb shaft N are supported as shown in figure below. Neglecting friction at contact surface P,Q,R and s, determine the reaction at R and S on shaft N.



(a)

From the F.B.D of M

$\sum Fy = 0 \uparrow +$

$Q \ \sin 40° - 300 = 0$

$Q = 467\ lb$ ∠40°

From the F.B.D of N

$\sum Fy = 0 \uparrow +$

$S - 500 - Q \ \sin 40° = 0$

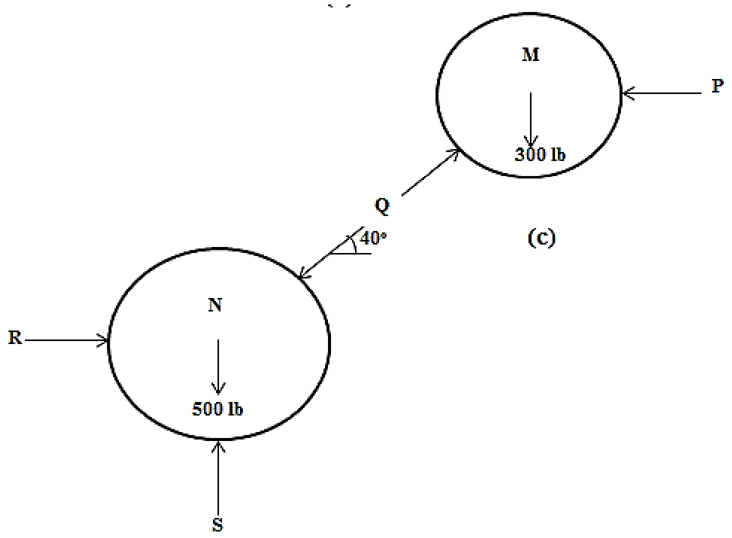
$S = 800\ lb \uparrow$

$\sum Fx = 0 \rightarrow$

$R - Q \cos 40° = 0$

$R = 467 \ \cos 40°$

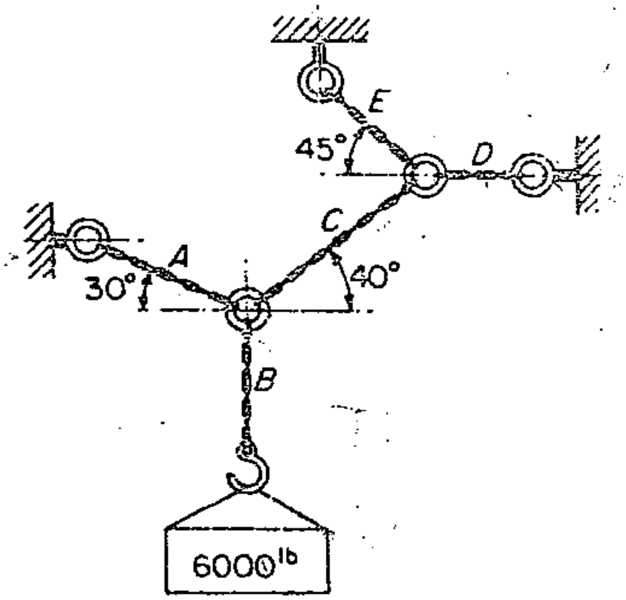
$R = 358\ lb$



(c)

(b)

Example: Determine the tensile force in chain D of the chain system shown in figure below.



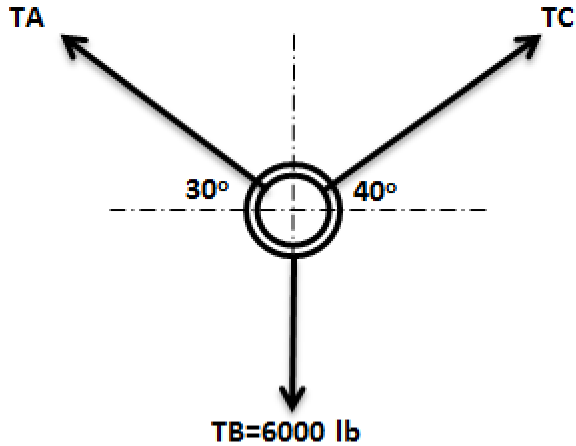


Fig (1)

From the F.B.D of fig (1)

$\sum Fx = 0 \ \rightarrow$

$T_C * \ \cos 40° - T_A * \cos 30 = 0$

$T_A = \ T_C \ * \frac{\cos 40°}{\cos 30°}$ ……………………(1)

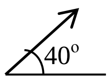
$\sum Fy = 0 \ \uparrow +$

$T_C * \sin 40° + T_A * \sin 30° - 6000 = 0$

$T_C * sin\, 40° + T_A * sin\, 30° = 6000$ ………(2)

Sub eq (1) in eq (2)

$T_C * sin\, 40° + T_C \ * sin\, 30° * \frac{cos\, 40°}{cos\, 30°} = 6000$

$T_C = 5530\ lb$ 

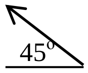
From the F.B.D of fig (2)

$\sum Fy = 0 \ \uparrow +$

$T_E * \sin 45° - \ T_C * \sin 40° = 0$

$$T_E = 5530 * \frac{sin\,40°}{sin\,45°}$$

$$T_E = 5027\ lb$$



$$\sum Fx = 0\ \rightarrow$$

$$-T_E * \cos 45° - T_C * \cos 40° + T_D = 0$$

$$T_D = 5027 * cos\,45° + 5530 * cos\,40°$$

$$T_D = 7791\ lb\ \rightarrow$$

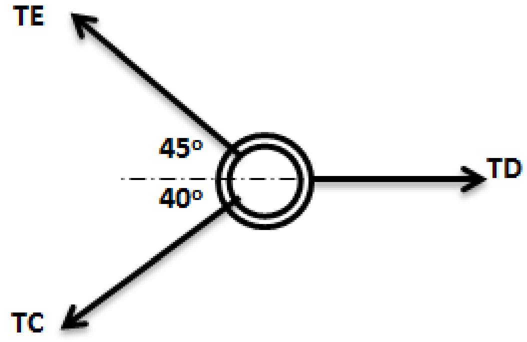
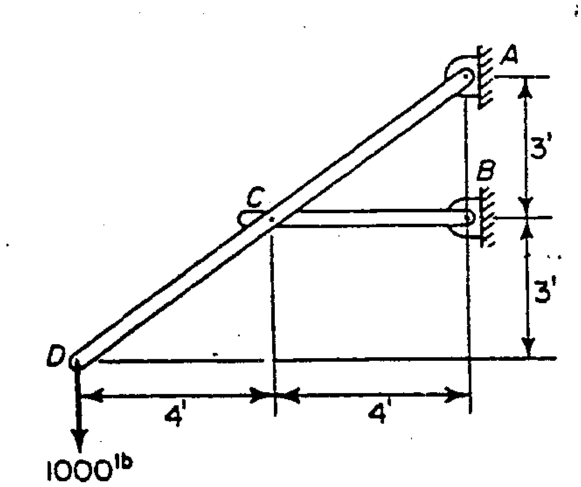
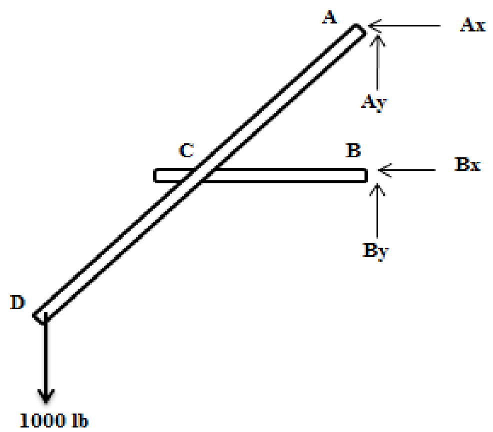


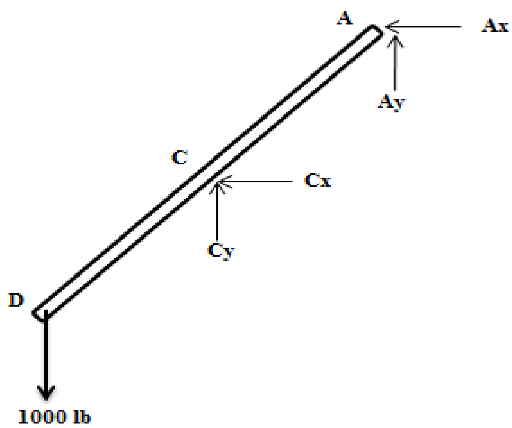
Fig (2)

Example: Determine the pin reaction at **A** on bar AD of figure below.





F.B.D for body



F.B.D for AD

F.B.D for whole body

$\sum MB = 0 + \circlearrowright$

$-Ax \ * 3 - 1000 * 8 = 0$

$Ax = \ -2667 \quad \therefore Ax = 2667 \ lb \ \rightarrow$

F.B.D for member **AD**

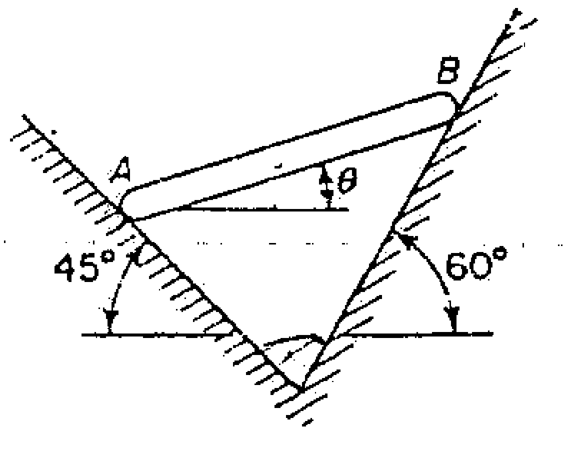
$\sum MC = 0 + \circlearrowright$

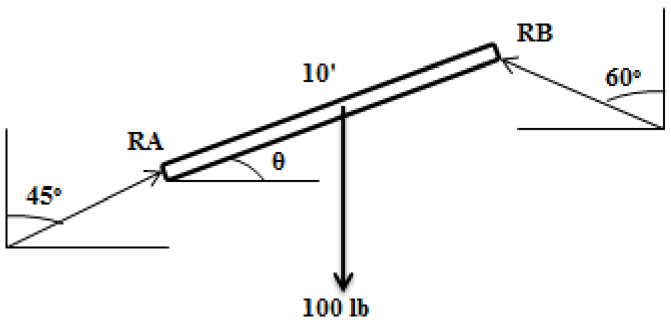
$Ax \ * 3 - Ay \ * 4 - 1000 * 4 = 0$

$2667 * 3 - Ay \ * 4 - 1000 * 4 = 0 \quad \rightarrow \ Ay = 1000 \ lb \ \uparrow$

$A = \ \sqrt{Ax^2 + Ay^2} = \ \sqrt{2667^2 + 1000^2} = 2848 \ lb$

Example: The uniform bar AB is 10 ft long and weighs 100 lb. The bar is placed with its ends on smooth inclined planes as shown in figure below. Determine the angle θ at which the bar will be in equilibrium.





F.B.D for Bar AB

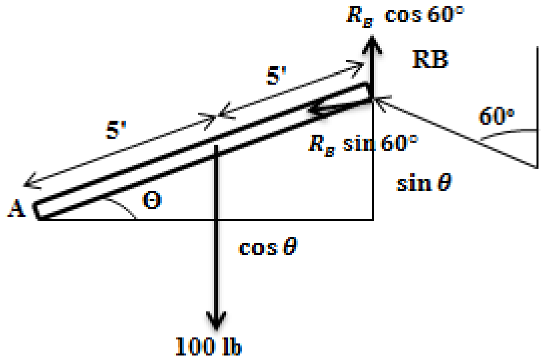
F.B.D for Bar AB

$\sum Fx = 0 \ \rightarrow$

$R_A \sin 45° - \ R_B \sin 60° = 0$

$R_A = 1.225 R_B$

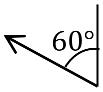
$\sum F_y = 0 \uparrow$



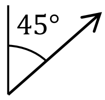
$R_A \cos 45° + R_B \cos 60° - 100 = 0$

$1.225 R_B \, cos\, 45° + R_B \, cos\, 60° = 100$

$R_B = 73.2\ lb$



$R_A = 1.225 * 73.2 = 89.7 lb$



$\sum MA = 0$ (+)

$R_B \, sin\, 60° * (10 \sin\theta) + R_B \, cos\, 60° * (10 \cos\theta) - 100 * (5 \cos\theta) = 0$

$73.2 * sin\, 60° * (10\ sin\,\theta) + 73.2 * cos\, 60° * (10\ cos\,\theta) - 100 * (5\, cos\,\theta) = 0$

$633.9\, sin\,\theta - 134\, cos\,\theta = 0$

$\frac{sin\,\theta}{cos\,\theta} = 0.2113 \rightarrow \tan\theta = 0.2113 \rightarrow \theta = \tan^{-1} 0.2113 \rightarrow \therefore\ \theta = 11.93°$