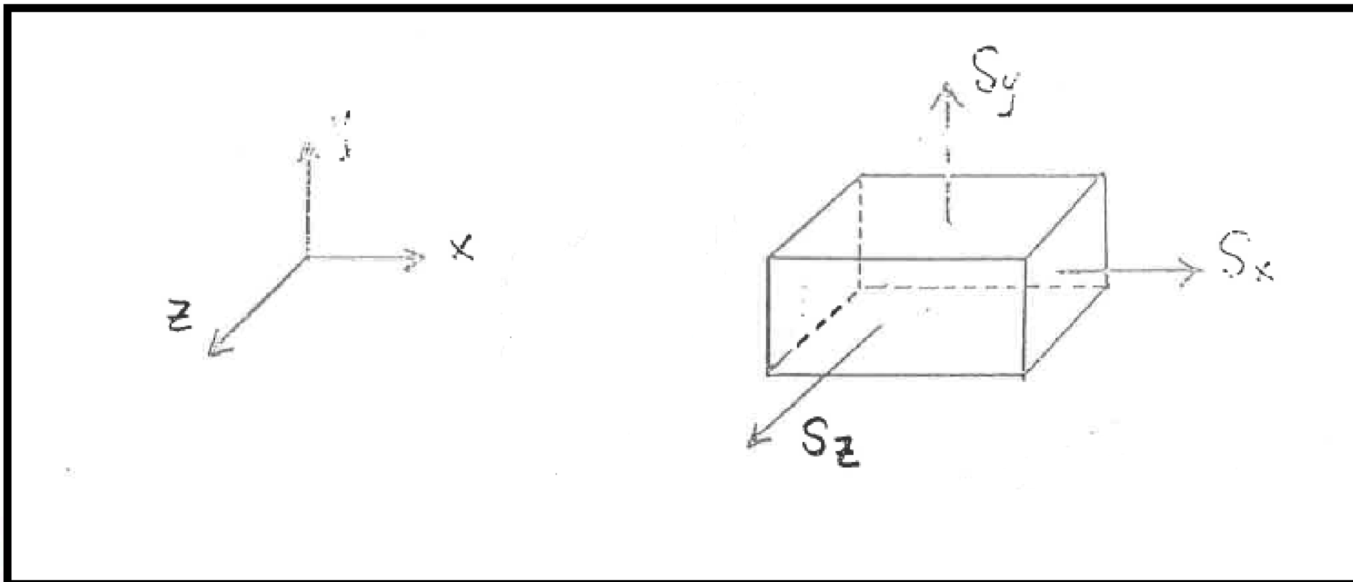
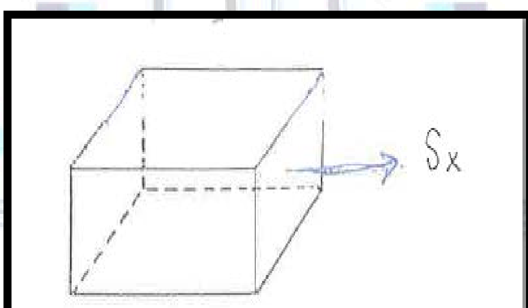
General expression for strain:-



Case (1) :-

Consider (اعتبر) Sx in X-direction only.

$$\textbf{\textit{Direct strain}}\ (\in_x) = \frac{\mathbf{Sx}}{\mathbf{E}}$$ **حفظ**



Induced strain due to x- stress :-

Induced strain in x-direction due to y-stress in y-direction :-

$$\mu = \frac{\epsilon_y}{\epsilon_x} \qquad \therefore\ \epsilon_y = \mu \,.\, \epsilon_x$$

Induced strain in Y-direction due to x-stress :-

$$= -\mu \,.\, \epsilon_x = -\mu \frac{Sx}{E}$$

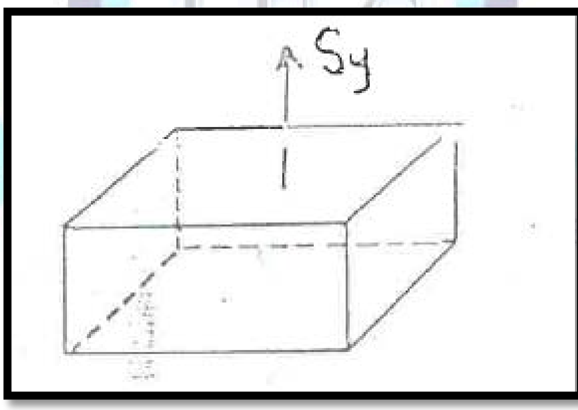
Induced strain in Z-direction due to x-stress :-

$$= -\mu \,.\, \epsilon_x = -\mu \frac{Sx}{E}$$

Case (2) :-

Consider Sy in Y-direction only

Direct strain in Y-direction $\textit{Direct strain}\ (\in_y) = \frac{Sy}{E}$ **حفظ**



Induced strain in X-direction due to Y-stress

$$= -\mu \frac{\mathbf{Sy}}{\mathbf{E}}$$

Induced strain in Y-direction due to Y-stress

$$= 0$$

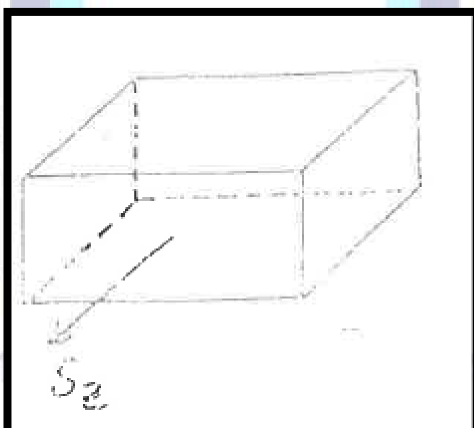
Induced strain in Z-direction due to Y-stress

$$= -\mu \frac{\mathbf{Sy}}{\mathbf{E}}$$

Case (3) :-

Consider Sz in Z-direction only

Direct strain in Y-direction $$\boldsymbol{Direct\ strain}\ (\in_z) = \frac{\mathbf{Sz}}{\mathbf{E}}$$ **حفظ**



Induced strain in X-direction due to Z-stress

$$= -\mu \frac{Sz}{E}$$

Induced strain in Y-direction due to Z-stress

$$= -\mu \frac{Sz}{E}$$

Induced strain in Z-direction due to Z-stress

$$= 0$$

Generalized Stresses مهم حفظ

Direction	Direct stress	Direct strain	Induced strain due to X-stress	Induced strain due to Y-stress	Induced strain due to Z-stress
X	Sx	$\frac{Sx}{E}$	0	$-\mu \frac{Sy}{E}$	$-\mu \frac{Sz}{E}$
Y	Sy	$\frac{Sy}{E}$	$-\mu \frac{Sx}{E}$	0	$-\mu \frac{Sz}{E}$
Z	Sz	$\frac{Sz}{E}$	$-\mu \frac{Sx}{E}$	$-\mu \frac{Sy}{E}$	0

Generalized Hooke's law Equations in Tension مهم حفظ

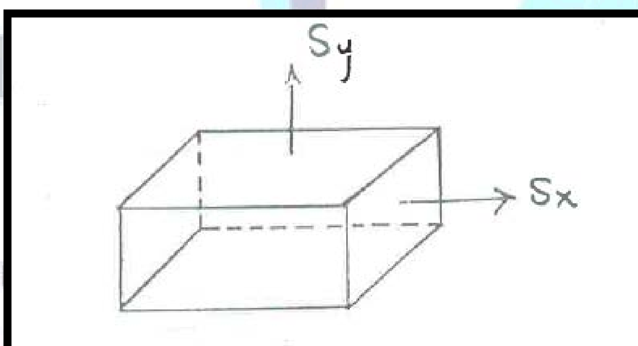
$$\epsilon_X = \frac{S_X}{E} - \mu \frac{Sy}{E} - \mu \frac{Sz}{E}$$

$$\epsilon_y = \frac{S_y}{E} - \mu \frac{Sx}{E} - \mu \frac{Sz}{E}$$

$$\epsilon_Z = \frac{S_Z}{E} - \mu \frac{Sx}{E} - \mu \frac{Sy}{E}$$

The generalized hooke's law equations for the case shown in fig. are :-

$$\epsilon_X = \frac{S_X}{E} - \mu \frac{Sy}{E}$$

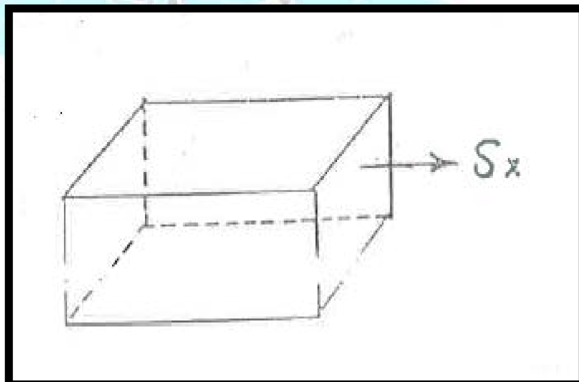


$$\epsilon_y = \frac{S_y}{E} - \mu \frac{Sx}{E}$$

$$\epsilon_Z = \frac{S_Z}{E} - \mu \frac{Sx}{E}$$

When the tensile stress affect in X-direction only the generalized hooke's law equations can be written as :-

$$\epsilon_X = \frac{S_X}{E}$$

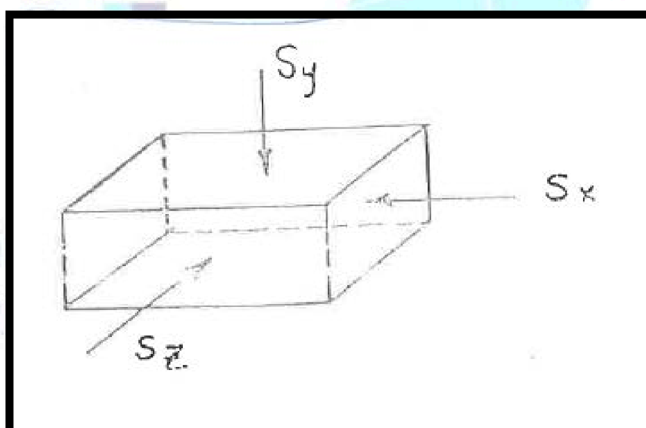


$$\epsilon_y = -\mu \frac{Sx}{E}$$

$$\epsilon_Z = -\mu \frac{Sx}{E}$$

Generalized Hooke's law Equations in Compression مهم حفظ

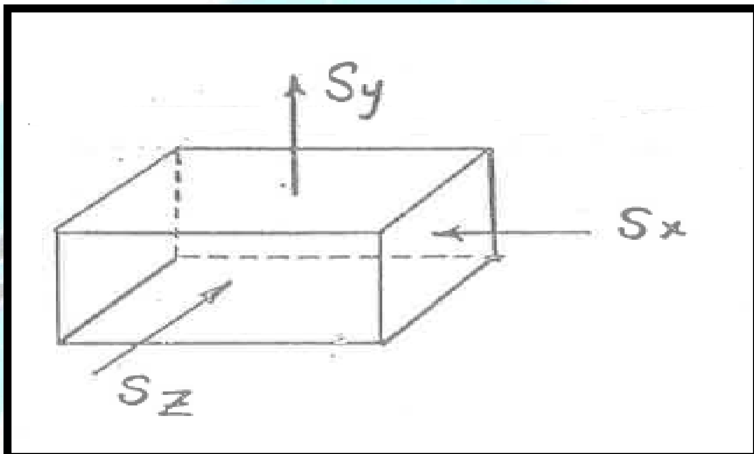
$$\epsilon_X = -\frac{S_X}{E} + \mu \frac{Sy}{E} + \mu \frac{Sz}{E}$$



$$\epsilon_y = -\frac{S_y}{E} + \mu \frac{Sx}{E} + \mu \frac{Sz}{E}$$

$$\epsilon_Z = -\frac{S_Z}{E} + \mu \frac{Sx}{E} + \mu \frac{Sy}{E}$$

H.W:- **Derive (اشتق) a generalized hooke's law equations for the state of stresses shown in fig. below .**



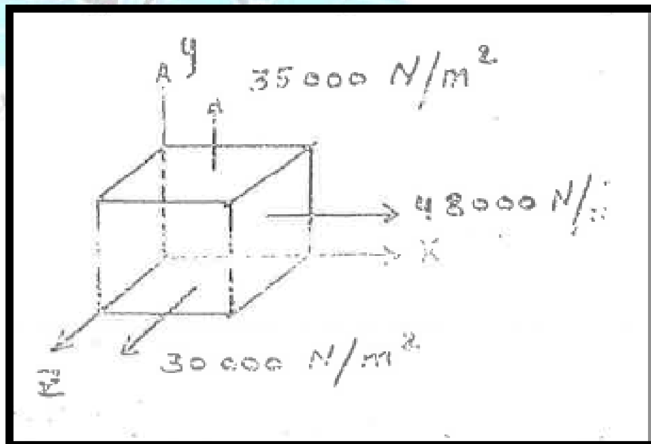
An important relation (علاقة مهمة) that connects (تربط) the three constants (الثوابت الثلاثة) (E,G, and μ) can be derived from theoretical considerations .The connecting equation is :-

$$G = \frac{E}{2(1+\mu)}$$ حفظ

Example (مهم شامل) :-

A cubical (مكعب) body is subjected to Sx= 73000 N/m^2 , Sy = 35000 N/m^2 , and Sz = 30000 N/m^2. if the length of the cube = 0.02 m , E=1x10^6 N/m^2 , μ = 0.25 . Determine the change in length in three directions.

Sol/



$$\epsilon_X = \frac{S_X}{E} - \mu\frac{Sy}{E} - \mu\frac{Sz}{E}$$

$$\frac{\delta_X}{X} = \frac{48000}{1\times10^6} - 0.25\frac{35000}{1\times10^6} - 0.25\frac{30000}{1\times10^6}$$

$$\frac{\delta_X}{0.02} = 0.048 - 0.00875 - 0.0075$$

$$\delta_X = 0.03175 \times 0.02 = 0.000635\, m = 0.635\, mm$$

$$\epsilon_y = \frac{S_y}{E} - \mu\frac{Sx}{E} - \mu\frac{Sz}{E}$$

$$\frac{\delta_y}{Y} = \frac{35000}{1\times10^6} - 0.25\frac{48000}{1\times10^6} - 0.25\frac{30000}{1\times10^6}$$

$$\frac{\delta_y}{0.02} = 0.035 - 0.012 - 0.0075$$

$$\delta_y = 0.0155 \times 0.02 = 0.00031\, m = 0.31\, mm$$

$$\epsilon_Z = \frac{S_Z}{E} - \mu \frac{Sx}{E} - \mu \frac{Sy}{E}$$

$$\frac{\delta_Z}{Z} = \frac{30000}{1 \times 10^6} - 0.25 \frac{48000}{1 \times 10^6} - 0.25 \frac{35000}{1 \times 10^6}$$

$$\frac{\delta_Z}{0.02} = 0.03 - 0.012 - 0.00875$$

$$\delta_Z = 0.00925 \times 0.02 = 0.000185\, m = 0.185\, mm$$